Pensieve Header: Verifying that the Nil-Hecke algebra acts on polynomials in n variables.

```
del[i_][f_] := Simplify[
  (f - (f /. {x[i] → x[i + 1], x[i + 1] → x[i]})) / (x[i] - x[i + 1])
 ]
```

```
del[1][f[x[1], x[2], x[3]]]
```

$$\frac{f[x[1], x[2], x[3]] - f[x[2], x[1], x[3]]}{x[1] - x[2]}$$

```
del[2][del[1][f[x[1], x[2], x[3]]]]
```

$$\frac{\frac{f[x[1],x[2],x[3]]-f[x[2],x[1],x[3]]}{x[1]-x[2]} + \frac{-f[x[1],x[3],x[2]]+f[x[3],x[1],x[2]]}{x[1]-x[3]}}{x[2] - x[3]}$$

```
lhs = del[1][del[2][del[1][f[x[1], x[2], x[3]]]]]
```

```
(f[x[1], x[2], x[3]] - f[x[1], x[3], x[2]] - f[x[2], x[1], x[3]] +
   f[x[2], x[3], x[1]] + f[x[3], x[1], x[2]] - f[x[3], x[2], x[1]]) /
 ((x[1] - x[2]) (x[1] - x[3]) (x[2] - x[3]))
```

```
rhs = del[2][del[1][del[2][f[x[1], x[2], x[3]]]]]
```

```
(f[x[1], x[2], x[3]] - f[x[1], x[3], x[2]] - f[x[2], x[1], x[3]] +
   f[x[2], x[3], x[1]] + f[x[3], x[1], x[2]] - f[x[3], x[2], x[1]]) /
 ((x[1] - x[2]) (x[1] - x[3]) (x[2] - x[3]))
```

```
lhs == rhs
```

```
True
```