

```

In[1]:= F[x_, y_, z_] := Det[
$$\begin{pmatrix} x[1] & x[2] & x[3] \\ y[1] & y[2] & y[3] \\ z[1] & z[2] & z[3] \end{pmatrix}$$
];
dot[x_, y_] := x[1] y[1] + x[2] y[2] + x[3] y[3];
Symmetrize[vars_, expr_] :=
  Sum[expr /. Thread[vars -> p], {p, Permutations[vars]}];
AntiSymmetrize[vars_, expr_] :=
  Sum[Signature[p] * expr /. Thread[vars -> p], {p, Permutations[vars]}];

In[5]:= Symmetrize[{a, b, c}, f[a, b, c]]
Out[5]= f[a, b, c] + f[a, c, b] + f[b, a, c] + f[b, c, a] + f[c, a, b] + f[c, b, a]

In[6]:= Factor[Symmetrize[{a, b, c, d, e, f}, F[a, b, c] F[d, e, f]]]
Out[6]= 0

In[7]:= Factor[Symmetrize[{a, b, c}, F[a, b, c] F[a, b, c]]]
Out[7]= 6 (a[3] b[2] c[1] - a[2] b[3] c[1] -
  a[3] b[1] c[2] + a[1] b[3] c[2] + a[2] b[1] c[3] - a[1] b[2] c[3])^2

In[8]:= Factor[AntiSymmetrize[{a, b, c}, dot[a, v1] dot[b, v2] dot[c, v3]]]
Out[8]= - (a[3] b[2] c[1] - a[2] b[3] c[1] -
  a[3] b[1] c[2] + a[1] b[3] c[2] + a[2] b[1] c[3] - a[1] b[2] c[3])
  (-v1[3] v2[2] v3[1] + v1[2] v2[3] v3[1] + v1[3] v2[1] v3[2] -
  v1[1] v2[3] v3[2] - v1[2] v2[1] v3[3] + v1[1] v2[2] v3[3])

In[9]:= Factor[AntiSymmetrize[{a, b, c, d}, dot[a, v1] dot[b, v2] dot[c, v3] dot[d, v4]]]
Out[9]= 0

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In[18]:= r1 = Factor[Symmetrize[{a, b, c, d}, F[a, b, c] dot[a, y1] dot[b, y2] dot[c, y3]]]
```

```
Out[18]= (a[3]^2 b[2]^2 c[1]^2 - 2 a[2] a[3] b[2] b[3] c[1]^2 + a[2]^2 b[3]^2 c[1]^2 -
  2 a[3]^2 b[1] b[2] c[1] c[2] + 2 a[2] a[3] b[1] b[3] c[1] c[2] +
  2 a[1] a[3] b[2] b[3] c[1] c[2] - 2 a[1] a[2] b[3]^2 c[1] c[2] +
  a[3]^2 b[1]^2 c[2]^2 - 2 a[1] a[3] b[1] b[3] c[2]^2 + a[1]^2 b[3]^2 c[2]^2 +
  2 a[2] a[3] b[1] b[2] c[1] c[3] - 2 a[1] a[3] b[2]^2 c[1] c[3] -
  2 a[2]^2 b[1] b[3] c[1] c[3] + 2 a[1] a[2] b[2] b[3] c[1] c[3] -
  2 a[2] a[3] b[1]^2 c[2] c[3] + 2 a[1] a[3] b[1] b[2] c[2] c[3] +
  2 a[1] a[2] b[1] b[3] c[2] c[3] - 2 a[1]^2 b[2] b[3] c[2] c[3] + a[2]^2 b[1]^2 c[3]^2 -
  2 a[1] a[2] b[1] b[2] c[3]^2 + a[1]^2 b[2]^2 c[3]^2 + a[3]^2 b[2]^2 d[1]^2 -
  2 a[2] a[3] b[2] b[3] d[1]^2 + a[2]^2 b[3]^2 d[1]^2 + a[3]^2 c[2]^2 d[1]^2 +
  b[3]^2 c[2]^2 d[1]^2 - 2 a[2] a[3] c[2] c[3] d[1]^2 - 2 b[2] b[3] c[2] c[3] d[1]^2 +
  a[2]^2 c[3]^2 d[1]^2 + b[2]^2 c[3]^2 d[1]^2 - 2 a[3]^2 b[1] b[2] d[1] d[2] +
  2 a[2] a[3] b[1] b[3] d[1] d[2] + 2 a[1] a[3] b[2] b[3] d[1] d[2] -
  2 a[1] a[2] b[3]^2 d[1] d[2] - 2 a[3]^2 c[1] c[2] d[1] d[2] - 2 b[3]^2 c[1] c[2] d[1] d[2] +
  2 a[2] a[3] c[1] c[3] d[1] d[2] + 2 b[2] b[3] c[1] c[3] d[1] d[2] +
  2 a[1] a[3] c[2] c[3] d[1] d[2] + 2 b[1] b[3] c[2] c[3] d[1] d[2] -
  2 a[1] a[2] c[3]^2 d[1] d[2] - 2 b[1] b[2] c[3]^2 d[1] d[2] + a[3]^2 b[1]^2 d[2]^2 -
  2 a[1] a[3] b[1] b[3] d[2]^2 + a[1]^2 b[3]^2 d[2]^2 + a[3]^2 c[1]^2 d[2]^2 + b[3]^2 c[1]^2 d[2]^2 -
  2 a[1] a[3] c[1] c[3] d[2]^2 - 2 b[1] b[3] c[1] c[3] d[2]^2 + a[1]^2 c[3]^2 d[2]^2 +
  b[1]^2 c[3]^2 d[2]^2 + 2 a[2] a[3] b[1] b[2] d[1] d[3] - 2 a[1] a[3] b[2]^2 d[1] d[3] -
  2 a[2]^2 b[1] b[3] d[1] d[3] + 2 a[1] a[2] b[2] b[3] d[1] d[3] +
  2 a[2] a[3] c[1] c[2] d[1] d[3] + 2 b[2] b[3] c[1] c[2] d[1] d[3] -
  2 a[1] a[3] c[2]^2 d[1] d[3] - 2 b[1] b[3] c[2]^2 d[1] d[3] - 2 a[2]^2 c[1] c[3] d[1] d[3] -
  2 b[2]^2 c[1] c[3] d[1] d[3] + 2 a[1] a[2] c[2] c[3] d[1] d[3] +
  2 b[1] b[2] c[2] c[3] d[1] d[3] - 2 a[2] a[3] b[1]^2 d[2] d[3] +
  2 a[1] a[3] b[1] b[2] d[2] d[3] + 2 a[1] a[2] b[1] b[3] d[2] d[3] -
  2 a[1]^2 b[2] b[3] d[2] d[3] - 2 a[2] a[3] c[1]^2 d[2] d[3] - 2 b[2] b[3] c[1]^2 d[2] d[3] +
  2 a[1] a[3] c[1] c[2] d[2] d[3] + 2 b[1] b[3] c[1] c[2] d[2] d[3] +
  2 a[1] a[2] c[1] c[3] d[2] d[3] + 2 b[1] b[2] c[1] c[3] d[2] d[3] -
  2 a[1]^2 c[2] c[3] d[2] d[3] - 2 b[1]^2 c[2] c[3] d[2] d[3] +
  a[2]^2 b[1]^2 d[3]^2 - 2 a[1] a[2] b[1] b[2] d[3]^2 + a[1]^2 b[2]^2 d[3]^2 +
  a[2]^2 c[1]^2 d[3]^2 + b[2]^2 c[1]^2 d[3]^2 - 2 a[1] a[2] c[1] c[2] d[3]^2 -
  2 b[1] b[2] c[1] c[2] d[3]^2 + a[1]^2 c[2]^2 d[3]^2 + b[1]^2 c[2]^2 d[3]^2)
(-y1[3] y2[2] y3[1] + y1[2] y2[3] y3[1] + y1[3] y2[1] y3[2] -
  y1[1] y2[3] y3[2] - y1[2] y2[1] y3[3] + y1[1] y2[2] y3[3])
```

```
In[13]:= r2 = Factor[
  Symmetrize[{a, b, c, d}, F[a, b, c] dot[a, y1] dot[b, y2] dot[c, d] dot[d, y3]]]
```

A very large output was generated. Here is a sample of it:

Out[13]=

$$\begin{aligned}
 & -a[3]^2 b[1] b[3] c[2]^2 d[1]^2 y1[2] y2[1] y3[1] - \\
 & a[1] a[3] b[3]^2 c[2]^2 d[1]^2 y1[2] y2[1] y3[1] - \\
 & a[3]^2 b[2]^2 c[1] c[3] d[1]^2 y1[2] y2[1] y3[1] + \\
 & \ll 1736 \gg + a[2]^2 b[1]^2 c[1] c[3] d[3]^2 y1[2] y2[3] y3[3] - \\
 & 2 a[1] a[2] b[1] b[2] c[1] c[3] d[3]^2 y1[2] y2[3] y3[3] + \\
 & a[1]^2 b[2]^2 c[1] c[3] d[3]^2 y1[2] y2[3] y3[3]
 \end{aligned}$$

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In[19]:= Simplify[r2 / r1]
```

A very large output was generated. Here is a sample of it:

$$\begin{aligned}
 & (a[1] a[2] (\ll 1 \gg) + \ll 3 \gg + a[3] \\
 & (a[1] (b[3]^2 (c[2] d[1] - c[1] d[2]) (c[2] (2 d[3] (-y1[2] y2[1] + y1[1] y2[2]) \\
 & y3[3] + d[1] (y1[2] y2[1] y3[1] - y1[1] y2[2] y3[1] + \\
 & y1[3] y2[2] y3[3] - y1[2] y2[3] y3[3])) + d[2] \\
 & (2 c[3] (y1[2] y2[1] - y1[1] y2[2]) y3[3] + c[1] (-y1[2] y2[1] y3[1] + \\
 & y1[1] y2[2] y3[1] - y1[3] y2[2] y3[3] + y1[2] y2[3] y3[3]))) + \\
 & b[1]^2 (\ll 1 \gg) (2 c[1] d[2] (-y1[3] y2[2] + y1[2] y2[3]) y3[1] + \\
 & \ll 1 \gg + c[2] (2 d[1] (y1[3] y2[2] - y1[2] y2[3]) y3[1] + d[3] (\ll 1 \gg))) - \\
 & \ll 1 \gg + \ll 1 \gg^2 (\ll 1 \gg) - 2 b[3] (b[1] (\ll 1 \gg) + b[2] (\ll 1 \gg))) + \ll 1 \gg)) / \\
 & ((a[1]^2 b[3]^2 c[2]^2 - 2 a[1]^2 b[2] b[3] c[2] c[3] + \\
 & a[1]^2 \\
 & b[2]^2 \\
 & c[3]^2 + \ll 39 \gg + \\
 & a[2]^2 (b[3]^2 (c[1]^2 + d[1]^2) + (\ll 1 \gg)^2 - \ll 1 \gg) + b[1]^2 (c[3]^2 + d[3]^2)) - \\
 & 2 \ll 3 \gg - \\
 & 2 a[3] \\
 & (a[2] (-b[1] b[3] (c[1] c[2] + d[1] d[2]) + (c[2] d[1] - c[1] d[2]) \\
 & (c[3] d[1] - c[1] d[3]) + b[1]^2 (c[2] c[3] + d[2] d[3]) + \\
 & b[2] (b[3] (c[1]^2 + d[1]^2) - b[1] (c[1] c[3] + d[1] d[3]))) + \\
 & a[1] (b[1] b[3] (c[2]^2 + d[2]^2) + (c[2] d[1] - c[1] d[2]) \\
 & (-c[3] d[2] + c[2] d[3]) + b[2]^2 (c[1] c[3] + d[1] d[3]) - b[2] \\
 & (b[3] (c[1] c[2] + d[1] d[2]) + b[1] (c[2] c[3] + d[2] d[3])))) (\ll 1 \gg))
 \end{aligned}$$

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```
In[20]:= Factor[Symmetrize[{a, b, c, d, e, f},
  F[a, b, c] dot[a, e] dot[e, y1] dot[b, f] dot[f, y2] dot[c, d] dot[d, y3]]]
```

Out[17]= \$Aborted

```
In[11]:= res1 = Expand[Symmetrize[{a, b, c, d, e, f},
  F[a, b, c] F[d, e, f] v1[a] v2[b] v3[c] v4[d] v5[e] v6[f]
]]
```

A very large output was generated. Here is a sample of it:

Out[11]=

```
<<38 879>> + a[1] b[2] c[3] d[1] e[2] f[3] v1[a] v2[b] v3[c] v4[d] v5[e] v6[f]
```

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In[12]:= res2 = Expand[Symmetrize[{a, b, c, d, e, f},
  dot[a, d] dot[b, e] dot[c, f] AntiSymmetrize[{d, e, f},
  v1[a] v2[b] v3[c] v4[d] v5[e] v6[f]
]]]
```

A very large output was generated. Here is a sample of it:

Out[12]=

```
<<38 879>> + a[1] b[2] c[3] d[1] e[2] f[3] v1[a] v2[b] v3[c] v4[d] v5[e] v6[f]
```

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In[13]:= res1 - res2
```

Out[13]= 0

```
In[14]:= res3 = Expand[Symmetrize[{a, b, c, d, e, f},
  F[d, e, f] F[d, e, f] dot[c, f] v1[a] v2[b] v3[c] v4[a] v5[b] v6[c]
]]
```

A very large output was generated. Here is a sample of it:

Out[14]=

```
2 a[1] d[1] d[3]^2 e[2]^2 f[1]^2 v1[c] v2[b] v3[a] v4[c] v5[b] v6[a] +
2 a[2] d[2] d[3]^2 e[2]^2 f[1]^2 v1[c] v2[b] v3[a] v4[c] v5[b] v6[a] +
2 a[3] d[3]^3 e[2]^2 f[1]^2 v1[c] v2[b] v3[a] v4[c] v5[b] v6[a] +
<<32 396>> + 2 a[1]^2 b[2]^2 c[3]^3 f[3] v1[d] v2[e] v3[f] v4[d] v5[e] v6[f]
```

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In[15]:= First /@ {res1, res2, res3}
```

```
Out[15]= {a[3] b[2] c[1] d[3] e[2] f[1] v1[f] v2[e] v3[d] v4[c] v5[b] v6[a],
  a[3] b[2] c[1] d[3] e[2] f[1] v1[f] v2[e] v3[d] v4[c] v5[b] v6[a],
  2 a[1] d[1] d[3]^2 e[2]^2 f[1]^2 v1[c] v2[b] v3[a] v4[c] v5[b] v6[a]}
```