```
F[x_, y_, z_] := Det[{{x[1], x[2], x[3]}, {y[1], y[2], y[3]}, {z[1], z[2], z[3]}}];
dot[x_, y_] := x[1] y[1] + x[2] y[2] + x[3] y[3];
Symmetrize[vars_, expr_] :=
  Sum[expr /. Thread[vars → p], {p, Permutations[vars]}];
AntiSymmetrize[vars_, expr_] :=
  Sum[Signature[p] * expr /. Thread[vars → p], {p, Permutations[vars]}];
```

```
Symmetrize[{a, b, c}, f[a, b, c]]
```

```
f[a, b, c] + f[a, c, b] + f[b, a, c] + f[b, c, a] + f[c, a, b] + f[c, b, a]
```

```
Factor[Symmetrize[{a, b, c, d, e, f}, F[a, b, c] F[d, e, f]]]
```

```
0
```

```
Factor[Symmetrize[{a, b, c}, F[a, b, c] F[a, b, c]]]
```

```
6 (a[3] b[2] c[1] - a[2] b[3] c[1] -
     a[3] b[1] c[2] + a[1] b[3] c[2] + a[2] b[1] c[3] - a[1] b[2] c[3])^2
```

```
False || Simplify[
  Symmetrize[{a, b, c, d, e, f}, F[a, b, c] F[d, e, f] dot[a, d] dot[b, e] dot[c, f]] /
   Symmetrize[{a, b, c}, F[a, b, c] F[a, b, c]]
 ]
```

Simplify::time :

Time spent on a transformation exceeded 300.` seconds, and the transformation was aborted. Increasing the value of TimeConstraint option may improve the result of simplification. ≫

```
$Aborted
```

```
Factor[AntiSymmetrize[{a, b, c}, dot[a, v1] dot[b, v2] dot[c, v3]]]
```

```
-(a[3] b[2] c[1] - a[2] b[3] c[1] -
     a[3] b[1] c[2] + a[1] b[3] c[2] + a[2] b[1] c[3] - a[1] b[2] c[3])
 (-v1[3] v2[2] v3[1] + v1[2] v2[3] v3[1] + v1[3] v2[1] v3[2] -
   v1[1] v2[3] v3[2] - v1[2] v2[1] v3[3] + v1[1] v2[2] v3[3])
```

```
Factor[AntiSymmetrize[{a, b, c, d}, dot[a, v1] dot[b, v2] dot[c, v3] dot[d, v4]]]
```

```
0
```

```
res1 = Expand[Symmetrize[{a, b, c, d, e, f},
   F[a, b, c] F[d, e, f] v1[a] v2[b] v3[c] v4[d] v5[e] v6[f]
  ]]
```

A very large output was generated. Here is a sample of it:

```
<<38 879>> + a[1] b[2] c[3] d[1] e[2] f[3] v1[a] v2[b] v3[c] v4[d] v5[e] v6[f]
```

Show Less | Show More | Show Full Output | Set Size Limit...

```
res2 = Expand[Symmetrize[{a, b, c, d, e, f},
    dot[a, d] dot[b, e] dot[c, f] AntiSymmetrize[{d, e, f},
      v1[a] v2[b] v3[c] v4[d] v5[e] v6[f]
     ]]]
```

A very large output was generated. Here is a sample of it:

```
<<38 879>> + a[1] b[2] c[3] d[1] e[2] f[3] v1[a] v2[b] v3[c] v4[d] v5[e] v6[f]
```

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```
res1 - res2
```

```
0
```

```
res3 = Expand[Symmetrize[{a, b, c, d, e, f},
    F[d, e, f] F[d, e, f] dot[c, f] v1[a] v2[b] v3[c] v4[a] v5[b] v6[c]
  ]]
```

A very large output was generated. Here is a sample of it:

```
2 a[1] d[1] d[3]^2 e[2]^2 f[1]^2 v1[c] v2[b] v3[a] v4[c] v5[b] v6[a] +
 2 a[2] d[2] d[3]^2 e[2]^2 f[1]^2 v1[c] v2[b] v3[a] v4[c] v5[b] v6[a] +
 2 a[3] d[3]^3 e[2]^2 f[1]^2 v1[c] v2[b] v3[a] v4[c] v5[b] v6[a] +
 <<32 396>> + 2 a[1]^2 b[2]^2 c[3]^3 f[3] v1[d] v2[e] v3[f] v4[d] v5[e] v6[f]
```

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```
First /@ {res1, res2, res3}
```

```
{a[3] b[2] c[1] d[3] e[2] f[1] v1[f] v2[e] v3[d] v4[c] v5[b] v6[a],
 a[3] b[2] c[1] d[3] e[2] f[1] v1[f] v2[e] v3[d] v4[c] v5[b] v6[a],
 2 a[1] d[1] d[3]^2 e[2]^2 f[1]^2 v1[c] v2[b] v3[a] v4[c] v5[b] v6[a]}
```