

```

F[x_, y_, z_] := Det[
$$\begin{pmatrix} x[1] & x[2] & x[3] \\ y[1] & y[2] & y[3] \\ z[1] & z[2] & z[3] \end{pmatrix}$$
]

dot[x_, y_] := x[1] y[1] + x[2] y[2] + x[3] y[3]

F[x, y, z] * F[u, v, z] - F[x, u, z] * F[y, v, z] + F[x, v, z] * F[y, u, z] // Simplify
0

Symmetrize[vars_, expr_] :=
  Sum[expr /. Thread[vars -> p], {p, Permutations[vars]}]

Symmetrize[{a, b, c}, f[a, b, c]]
f[a, b, c] + f[a, c, b] + f[b, a, c] + f[b, c, a] + f[c, a, b] + f[c, b, a]

Factor[Symmetrize[{a, b, c, d, e, f}, F[a, b, c] F[d, e, f]]]
0

Factor[Symmetrize[{a, b, c}, F[a, b, c] F[a, b, c]]]
6 (a[3] b[2] c[1] - a[2] b[3] c[1] -
  a[3] b[1] c[2] + a[1] b[3] c[2] + a[2] b[1] c[3] - a[1] b[2] c[3])^2

res1 = Expand[Symmetrize[{a, b, c, d, e, f},
  F[a, b, c] F[d, e, f] v1[a] v2[b] v3[c] v4[d] v5[e] v6[f]
]]

```

A very large output was generated. Here is a sample of it:

```
<<38 879>> + a[1] b[2] c[3] d[1] e[2] f[3] v1[a] v2[b] v3[c] v4[d] v5[e] v6[f]
```

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```

res2 = Expand[Symmetrize[{a, b, c, d, e, f},
  dot[a, d] dot[b, e] dot[c, f] v1[a] v2[b] v3[c] v4[d] v5[e] v6[f]
]]

```

A very large output was generated. Here is a sample of it:

```

a[1] b[1] c[1] d[1] e[1] f[1] v1[f] v2[e] v3[d] v4[c] v5[b] v6[a] +
a[2] b[1] c[1] d[2] e[1] f[1] v1[f] v2[e] v3[d] v4[c] v5[b] v6[a] + <<19 436>> +
a[2] b[3] c[3] d[2] e[3] f[3] v1[a] v2[b] v3[c] v4[d] v5[e] v6[f] +
a[3] b[3] c[3] d[3] e[3] f[3] v1[a] v2[b] v3[c] v4[d] v5[e] v6[f]

```

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```
res3 = Expand[Symmetrize[{a, b, c, d, e, f},
  F[d, e, f] F[d, e, f] dot[c, f] v1[a] v2[b] v3[c] v4[a] v5[b] v6[c]
]]
```

A very large output was generated. Here is a sample of it:

```
2 a[1] d[1] d[3]^2 e[2]^2 f[1]^2 v1[c] v2[b] v3[a] v4[c] v5[b] v6[a] +
2 a[2] d[2] d[3]^2 e[2]^2 f[1]^2 v1[c] v2[b] v3[a] v4[c] v5[b] v6[a] +
2 a[3] d[3]^3 e[2]^2 f[1]^2 v1[c] v2[b] v3[a] v4[c] v5[b] v6[a] +
<<32396>> + 2 a[1]^2 b[2]^2 c[3]^3 f[3] v1[d] v2[e] v3[f] v4[d] v5[e] v6[f]
```


First /@ {res1, res2, res3}

```
{a[3] b[2] c[1] d[3] e[2] f[1] v1[f] v2[e] v3[d] v4[c] v5[b] v6[a],
 a[1] b[1] c[1] d[1] e[1] f[1] v1[f] v2[e] v3[d] v4[c] v5[b] v6[a],
 2 a[1] d[1] d[3]^2 e[2]^2 f[1]^2 v1[c] v2[b] v3[a] v4[c] v5[b] v6[a]}
```