Once more, let's try to get from braids to virtual braids in an abstract way.

B: Braids (& lin. combinations thereof)

I: The ideal generated by $\times - \times$.

Consider

$$F\left(\frac{B}{I} \oplus I\right) \Bigg/ \begin{array}{l} \text{Ker } \mu_{00} : \frac{B}{I} \otimes \frac{B}{I} \to \frac{B}{I} \\ \text{Ker } \mu_{01} : \frac{B}{I} \otimes I \to I \end{array} \, ?$$