Multicategories

April-23-11
11:40 AM

From http://en.wikipedia.org/wiki/Multicategory:

Definition

[edit]

A multicategory consists of

- a collection (often a proper class) of *objects*;
- for every finite sequence $(X_1, X_2, \ldots, X_n)$ of objects (for $n := 0, 1, 2, \ldots$) and object Y, a set of *morphisms* from $X_1, X_2, \ldots$, and X_n to Y; and
- for every object X, a special identity morphism (with $n := 1$) from X to X.

Additionally, there are composition operations: Given a sequence of sequences $(X_{1,1}, X_{1,2}, \ldots, X_{1,n1}; X_{2,1}, X_{2,2}, \ldots, X_{2,n2}; \ldots; X_{m,1}, X_{m,2}, \ldots, X_{m,nm})$ of objects, a sequence $(Y_1, Y_2, \ldots, Y_m)$ of objects, and an object Z: if

- f_1 is a morphism from $X_{1,1}, X_{1,2}, \ldots$, and $X_{1,n}$ to Y_1;
- f_2 is a morphism from $X_{2,1}, X_{2,2}, \ldots$, and $X_{2,n}$ to Y_2;
- ...;
- f_m is a morphism from $X_{m,1}, X_{m,2}, \ldots$, and $X_{m,n}$ to Y_m; and
- g is a morphism from $Y_1, Y_2, \ldots$, and Y_m to Z:

then there is a composite morphism $g(f_1, f_2, \ldots, f_m)$ from $X_{1,1}, X_{1,2}, \ldots, X_{1,n1}, X_{2,1}, X_{2,2}, \ldots, X_{2,n2}, \ldots, X_{m,1}, X_{m,2}, \ldots$, and $X_{m,nm}$ to Z. This must satisfy certain axioms:

- If m is 1, Z is Y, and g is the identity morphism for Y, then $g(f)$ must equal f;
- if n_1 is 1, n_2 is 1, ..., n_m is 1, X_1 is Y_1, X_2 is Y_2, ..., X_m is Y_m, f_1 is the identity morphism for Y_1, f_2 is the identity morphism for Y_2, ..., and f_m is the identity morphism for Y_m, then $g(f_1, f_2, \ldots, f_m)$ must equal g; and
- an associativity condition (involving a further level of composition) that takes a long time to write down.

A multicategory of sets & functions is exactly what I call an "algebraic structure". } I.e., a functor from some given multicategory into the m.c. of sets & functions

Weaknesses: 1. No cloning operations.

~~(2. No built-in axioms & "free" objects)~~

3. No "units".