

2D Extended TQFT \rightarrow 2-Category Alg:

Objects \leftrightarrow algebras

Morphisms: $\{A_1 \rightarrow A_2\} = \{ \begin{smallmatrix} A_1-A_2 \text{ bimodules} \\ A_1 \mathcal{M}_{A_2} \end{smallmatrix} \}$

2-Morphisms: morphisms of bimodules.

$Z(\bullet) = A$, a finite-dimensional semi-simple alg.,
along w/ $\epsilon: A \rightarrow \mathbb{C}$ sit.

$(a, b) := \epsilon(ab)$ is symmetric and non degenerate.

$$Z(\bullet \longrightarrow \bullet) = {}_A A_A$$

$Z(\bigcirc) =$ the centre of A .

$$Z(\text{cap}) = \text{id}_{{}_A A_A}$$

$$Z(\text{triangle}) = ({}_A A_A \otimes_{{}_A A_A} \xrightarrow{m} {}_A A_A)$$

Mon: a 3-category with
Objects Monoidal Categories

1-morphisms: $A_i - A_k$ bimodule categories:

$$\begin{aligned} A_i \otimes M_j &\rightarrow M_j \\ M_j \otimes A_i &\rightarrow M_j \end{aligned} \quad (\text{Functors})$$

2-morphisms: Functors ${}_{A_i} \mathcal{M}_{A_k} \rightarrow {}_{A_i} \mathcal{M}_{A_l}$ compatible
with bi-module structure.

3-morphisms: Functorial morphisms (nat trans?)

Theorem. Every "Spherical Category" $\mathcal{A} (\otimes, *)$
defines an extended 3d TQFT w/ values

This goes into making Turaev-Viro into a 3-category valued invariant.

in Mon, via

$$Z(\circ) = A \quad Z(\circ \rightarrow \circ) = \underset{A}{A} \underset{A}{A} \quad \text{etc.}$$