Algebraic Structures

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Goal - find the most general context in which the following makes sense:

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Abstract Generalities. (R, I): an algebra and an "augmentation ideal" in it. $\hat{R} := \varprojlim R/I^m$ the "I-adic completion". $\operatorname{gr}_I R := \widehat{\bigoplus} I^m/I^{m+1}$ has a product μ, especially, $\mu_{11}\colon (C = I/I^2)^{\otimes 2} \to I^2/I^3$. The "quadratic approximation" $\mathcal{A}_I(R) := FC/\langle \ker \mu_{11} \rangle$ of R surjects using μ on $\operatorname{gr} R$.

The Prized Object. A "homomorphic $\mathcal{A}$-expansion": a homomorphic filtered $Z\colon R \to \mathcal{A}$ inducing the identity on $I/I^2 = C$.

From swiss knots - 1105 handout.

See also 2009-10: Antolin@GSS: What is algebra, really?
Look also at "Lawvere".

Baby version: From non-pure braids to $\mathcal{A}^b$.

Challenge. Is there any "mechanical" way to get from the planar algebra of tangles to the planar/circuit algebra of (unshielded) chord diagrams?

Solution. Take

$$F\left(\tfrac{K}{I} \oplus \tfrac{I}{I^2}\right) \Big/ \left\langle \begin{array}{l} \ker \mu_{00}: K/I \otimes K/I \to K/I \\ \ker \mu_{01}: K/I \otimes I/I^2 \to I/I^2 \\ \ker \mu_{11}: I/I^2 \otimes I/I^2 \to I^2/I^3 \end{array} \right\rangle$$

My current best is to use multicategories; there is a fair description within of "free objects" and "ideals", though I'm not sure what to do about cloning, and if at all anything needs to be done.