Sinai@FI: Moebius Function and Statistical Mechanics

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$$\zeta(1+is_2) = \sum_{n\geq 1} \frac{1}{n^{1+is_2}}$$ does not converge

$$\frac{1}{n^{1+is}} = e^{-(1+is_2)\log n}$$

The Möbius function:

$$\mu(n) = \begin{cases} 1 & n=1 \\ 0 & n \text{ isn't square free} \\ (-1)^m & n = \prod_{i=1}^{m} p_i \end{cases}$$

$$\zeta(s)^{-1} = \sum \frac{\mu(n)}{n^s}$$ (at least for $Re(s)>1$, but also for $Re(s)=1$)

$$N_n = |\{m<n \text{ is square-free}\}| \sim c\cdot n$$

1. $\sum_{n=1}^{N} \mu(n) = o(N) \Longrightarrow$ prime number thm.

2. $\sum_{n\leq N} \mu(n) \leq C_\epsilon N^{\frac{1}{2}+\epsilon} \Longrightarrow$ Riemann Hypothesis.

Let $\Omega_m = \{p_1^{\nu_1}\cdot p_2^{\nu_2}\cdot \ldots \cdot p_m^{\nu_m} : \nu_i \in \{0,1\}\}$.