FullToPrimitives from Pensieve/2009-12:

```
PrimitivesToFull[p_List] := Module[
   {lp, x, ser},
   lp = Length[p];
   ser = Normal[Series[
      Product[(1 - x^i)^(-p[[i]]), {i, lp}],
      {x, 0, lp}
     ]];
   Table[Coefficient[ser, x, i], {i, 0, lp}]
  ];
FullToPrimitives[{1}] = {};
FullToPrimitives[{1, mid___, last_}] := Module[{prev},
  prev = FullToPrimitives[{1, mid}];
  Append[
   prev,
   last - Last[PrimitivesToFull[Append[prev, 0]]]
  ]
 ]
```

```
FullToPrimitives[Table[2^k, {k, 0, 10}]]
```

```
{2, 1, 2, 3, 6, 9, 18, 30, 56, 99}
```

{2,1,2,3,6,9,18,30,56,99}

```
{2, 1, 2, 3, 6, 9, 18, 30, 56, 99}
```

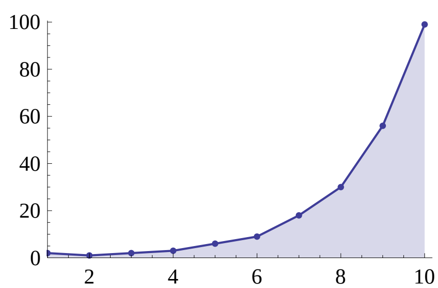
Input:

```
{2, 1, 2, 3, 6, 9, 18, 30, 56, 99}
```

{2, 1, 2, 3, 6, 9, 18, 30, 56, 99}

Plot:

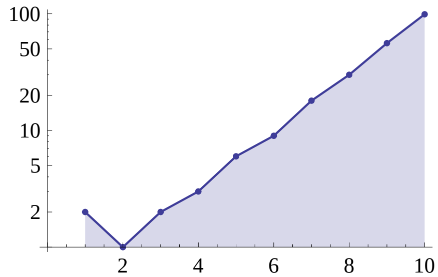
```
ListLinePlot[{2, 1, 2, 3, 6, 9, 18, 30, 56, 99},
 Mesh -> All, Filling -> Axis, AxesOrigin -> {1, 0}]
```



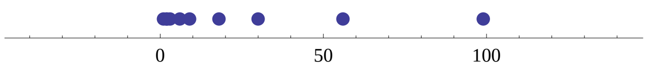
Log–linear plot:

Log–linear plot:

```
ListLogPlot[{2, 1, 2, 3, 6, 9, 18, 30, 56, 99},
 Joined -> True, Mesh -> All, Filling -> Axis]
```



Number line:



Length of data:

```
Length[{2, 1, 2, 3, 6, 9, 18, 30, 56, 99}]
```

10 items

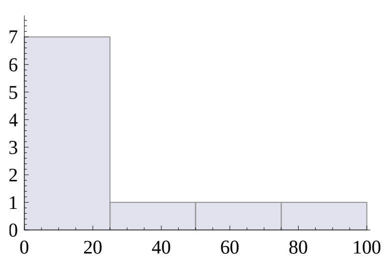
Total:

```
Total[{2, 1, 2, 3, 6, 9, 18, 30, 56, 99}]
```

$2+1+2+3+6+9+18+30+56+99=226$

Histogram:

```
Histogram[{2, 1, 2, 3, 6, 9, 18, 30, 56, 99}]
```



Statistics:

More

mean	22.6

median	7.5
sample standard deviation	31.95

Successive ratios:

Exact form

```
N[Rest[{2, 1, 2, 3, 6, 9, 18, 30, 56, 99}] /
  Most[{2, 1, 2, 3, 6, 9, 18, 30, 56, 99}]]
```

$\frac{1}{2}$, 2, 1.5, 2, 1.5, 2, 1.66667, 1.86667, 1.76786

Diophantine relations:

$$2+1+2+3+6-9-18-30-56+99=0$$

$$2+1+2-3-6+9-18-30-56+99=0$$

{2, 1, 2, 3, 6, 9, 18, 30, 56, 99}

```
Import["http://oeis.org/search?q=2,1,2,3,6,9,18,30,56,99"]
```

login
This site is supported by donations to The OEIS Foundation .

Hints
(Greetings from The On-Line Encyclopedia of Integer Sequences !)   Search: seq:2,1,2,3,6,9,18,30,56,99

Displaying 1-1 of 1 result found.   page 1
Sort: relevance | references | number

A001037   Number of degree-n irreducible polynomials over GF(2); number of n-bead necklaces with beads of 2 colors when turning over is not allowed and with primitive period n; number of binary Lyndon words of length n.
(Formerly M0116 N0046)   +20
67

1, 2 , 1 , 2 , 3 , 6 , 9 , 18 , 30 , 56 , 99 , 186, 335,
630, 1161, 2182, 4080, 7710, 14532, 27594, 52377, 99858, 190557,
364722, 698870, 1342176, 2580795, 4971008, 9586395, 18512790,
35790267, 69273666, 134215680, 260300986, 505286415, 981706806
(list ; graph ; listen ; history ; internal format)

OFFSET
0,2
COMMENTS
Also dimensions of free Lie algebras -
see A059966 , which is essentially the same sequence.
This sequence also represents the number N of cycles of length L
in a digraph under x^2 seen modulo a Mersenne prime M_q=2^q-1.
This number does not depend on q and L is any divisor of q-1.
See Theorem 5 and Corollary 3 of the Shallit and Vasiga paper:
N=sum(eulerphi(d)/order(d,2)) where d is a divisor of 2^(q-1)-1 such
that order(d,2)=L. - Tony Reix (Tony.Reix(AT)laposte.net), Nov 17 2005
Except for a(0) = 1, Bau-Sen Du's [1985/2007] Table 1, p. 6, has
this sequence as the 7th (rightmost) column. Other columns of
the table include (but are not identified as) A006206 - A006208
. - Jonathan Vos Post (jvospost3(AT)gmail.com), Jun 18 2007
"Number of binary Lyndon words" means: number of binary strings inequivalent
modulo rotation (cyclic permutation) of the digits and not having a
period smaller than n. This provides a link to A103314 , since these
strings correspond to the inequivalent zero-sum subsets of U_m (m-th
roots of unity) obtained by taking the union of U_n (n|m) with 0 or more
U_d (n | d, d | m) multiplied by some power of exp(i 2pi/n) to make them
mutually disjoint. (But not all zero-sum subsets of U_m are of that
form.) - M. F. Hasler (Maximilian.Hasler(AT)gmail.com), Jan 14 2007
Contribution from Mathilde Noual (mathilde.noual(AT)ens-lyon.fr),
Feb 25 2009: (Start)
Also the number of dynamical cycles of period
n of a threshold Boolean automata
network which is a quasi-minimal positive circuit of size a
multiple of n and which is updated in parallel. (End)
Contribution from Pietro Majer (majer(AT)dm.unipi.it), Sep 22 2009: (Start)
Also, the number of periodic points with (minimal) period n in the iteration
of the tent map f(x):=2min{x,1-x} on the unit interval. (End)
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[From Mathilde Noual (mathilde.noual(AT)ens-lyon.fr), Feb 25 2009]

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4086/2006, Springer-Verlag. [From N. J. A. Sloane, Jul 09 2009]
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periodic orbits , J. Integer Seqs., Vol. 4 (2001), #01.2.1.
F. Ruskey, Necklaces, Lyndon words, De Bruijn sequences, etc.
F. Ruskey, Primitive and Irreducible Polynomials
Eric Weisstein's World of Mathematics, Irreducible Polynomial
Eric Weisstein's World of Mathematics, Lyndon Word
Wikipedia, Lyndon word
Index entries for sequences related to Lyndon words
Index entries for "core" sequences

FORMULA

a(n) = (1/n) sum_{ d divides n } mu(n/d) 2^d.
A000031 (n) = sum_{ d divides n }
A001037 (d); 2^n = sum_{ d divides n } d* A001037 (d).
G.f.: 1 - Sum_{n>=1} moebius(n)*log(1 - 2*x^n)/n, where moebius(n)=

A008683 (n). [From Paul D. Hanna (pauldhanna(AT)juno.com), Oct 13 2010]
EXAMPLE
Binary strings (Lyndon words): a(0) = 1 = #{ "" },
a(1) = 2 = #{ "0", "1" }, a(2) = 1 = #{ "01" }, a(3) = 2 = #{
"001", "011" }, a(4) = 3 = #{ "0001", "0011", "0111" }, a(5) =
6 = #{ "00001", "00011", "00101", "00111", "01011", "01111" }
MAPLE
with(numtheory): A001037 := proc(n) local a, d;
if n = 0 then RETURN(1); else a := 0: for d in divisors(n)
do a := a+mobius(n/d)*2^d; od: RETURN(a/n); fi; end;
MATHEMATICA
Table[Apply[Plus, MoebiusMu[n
/ Divisors[n]]*2^Divisors[n]]/n, {n, 1, 32}]
PROG
(PARI) a(n)=if(n<1, n==0, sumdiv(n, d, moebius(d)*2^(n/d))/n)
(PARI) {a(n)=polcoeff(1-sum(k=1, n, moebius(k)/k*log(1-2*x^k+x*O(x^n))),
n)} [From Paul D. Hanna (pauldhanna(AT)juno.com), Oct 13 2010]
CROSSREFS
See A058943 and A102569 for initial
terms. See also A058947 , A011260 , A059966 .
Irreducible over GF(2), GF(3), GF(4), GF(5), GF(7): A058943 , A058944 ,
A058948 , A058945 , A058946 . Primitive irreducible over GF(2), GF(3),
GF(4), GF(5), GF(7): A058947 , A058949 , A058952 , A058950 , A058951 .
Cf. A000031 (n-bead necklaces but may have period dividing n), A014580 ,
A046211 , A046209 . Equals A000048 + A051841 . Also equals A027375 (n)/n.
Euler transform is A000079 .
Cf. A006206 - A006208 , A038063 , A060477 .
Cf. A103314 ; A059966 (n)= A060477 (n)= A001037 (n) for all n>1.
KEYWORD
nonn , core , easy , nice
AUTHOR
N. J. A. Sloane (njas(AT)research.att.com).
EXTENSIONS
Replace arXiv URL by non-cached version - R.
J. Mathar (mathar(AT)strw.leidenuniv.nl), Oct 23 2009


```
FullToPrimitives[Table[3^k, {k, 0, 10}]]
```

```
{3, 3, 8, 18, 48, 116, 312, 810, 2184, 5880}
```

3,3,8,18,48,116,312,810,2184,5880

```
{3, 3, 8, 18, 48, 116, 312, 810, 2184, 5880}
```

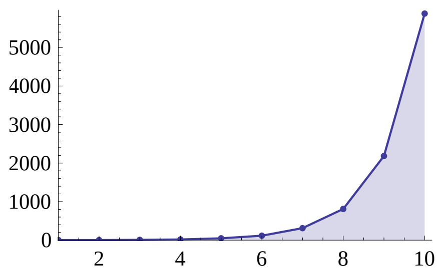
Input:

```
{3, 3, 8, 18, 48, 116, 312, 810, 2184, 5880}
```

{3, 3, 8, 18, 48, 116, 312, 810, 2184, 5880}

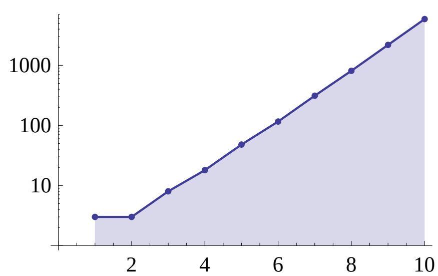
Plot:

```
ListLinePlot[{3, 3, 8, 18, 48, 116, 312, 810, 2184, 5880},
 Mesh -> All, Filling -> Axis, AxesOrigin -> {1, 0}]
```

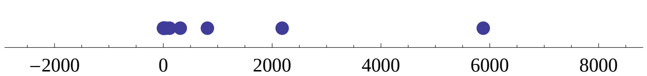


Log–linear plot:

```
ListLogPlot[{3, 3, 8, 18, 48, 116, 312, 810, 2184, 5880},
 Joined -> True, Mesh -> All, Filling -> Axis]
```



Number line:



Length of data:

```
Length[{3, 3, 8, 18, 48, 116, 312, 810, 2184, 5880}]
```

10 items

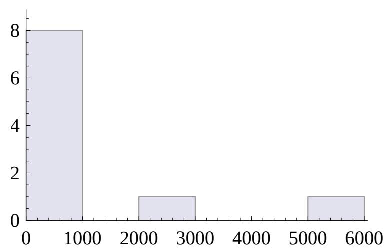
Total:

```
Total[{3, 3, 8, 18, 48, 116, 312, 810, 2184, 5880}]
```

$3+3+8+18+48+116+312+810+2184+5880=9382$

Histogram:

```
Histogram[{3, 3, 8, 18, 48, 116, 312, 810, 2184, 5880}]
```



Statistics:

More

mean	938.2
median	82
sample standard deviation	1865

Successive ratios:

Exact form

```
N[Rest[{3, 3, 8, 18, 48, 116, 312, 810, 2184, 5880}] /
  Most[{3, 3, 8, 18, 48, 116, 312, 810, 2184, 5880}]]
```

1, 2.66667, 2.25, 2.66667, 2.41667, 2.68966, 2.59615, 2.6963, 2.69231

```
{3, 3, 8, 18, 48, 116, 312, 810, 2184, 5880}
```

```
Import["http://oeis.org/search?q=3,3,8,18,48,116,312,810,2184,5880"]
```

```
login
      This site is supported by donations to The OEIS Foundation .
```

Hints
(Greetings from The On-Line Encyclopedia of Integer Sequences !) Search: seq:3,3,8,18,48,116,312,810,2184,5880

Displaying 1-1 of 1 result found. page 1
Sort: relevance | references | number

A027376 Number of ternary irreducible polynomials of degree n; dimensions of free Lie algebras. +20
23

1, 3 , 3 , 8 , 18 , 48 , 116 , 312 , 810 , 2184 , 5880 , 16104, 44220, 122640, 341484, 956576, 2690010, 7596480, 21522228, 61171656, 174336264, 498111952, 1426403748, 4093181688, 11767874940, 33891544368, 97764009000, 282429535752 (list ; graph ; listen ; history ; internal format)

OFFSET
0,2
COMMENTS
Number of Lyndon words of length n on {1,2,3}. A Lyndon word is primitive (not a power of another word) and is earlier in lexicographic order than any of its cyclic shifts. - John W. Layman (layman(AT)math.vt.edu), Jan 24 2006
Exponents in an expansion of the Hardy-Littlewood constant product(1-(3*p-1)/(p-1)^3, p prime >= 5), whose decimal expansion is in A065418 : the constant equals product_{n>=2} (zeta(n)*(1-2^-n)*(1-3^-n))^-a(n). - Michael Somos, Apr 05 2003
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E. N. Gilbert and J. Riordan, Symmetry types of periodic sequences, Illinois J. Math., 5 (1961), 657-665.
M. Lothaire, Combinatorics on Words. Addison-Wesley, Reading, MA, 1983, p. 79.
G. Viennot, Algebres de Lie Libres et Monoides Libres, Lecture Notes in Mathematics 691, Springer verlag 1978.
LINKS
T. D. Noe, Table of n, a(n) for n=0..200
Y. Puri and T. Ward, Arithmetic and growth of periodic orbits , J. Integer Seqs., Vol. 4 (2001), #01.2.1.
G. Niklasch, Some number theoretical constants: 1000-digit values [Cached copy]
Index entries for sequences related to Lyndon words
FORMULA
Sum mu(d)*3^(n/d)/n; d|n. (1-3x)=Product_{n>0} (1-x^n)^a(n).

MAPLE
```
A027376 := proc(n) local d, s; if n = 0 then RETURN(1); else s := 0; for d in
  divisors(n) do s := s+mobius(d)*3^(n/d); od; RETURN(s/n); fi; end;
```
MATHEMATICA
```
a[0]=1; a[n_] := Module[{ds=Divisors[n], i},
  Sum[MoebiusMu[ds[[i]]]3^(n/ds[[i]]), {i, 1, Length[ds]}]/n]
```
PROG
```
(PARI) a(n)=if(n<1, n==0, sumdiv(n, d, moebius(n/d)*3^d)/n)
```
CROSSREFS

Cf. A001693 , A000031 , A001037 ,
A027375 , A027377 , A054718 , A001867 , A102660 .

KEYWORD

nonn , nice , easy

AUTHOR

N. J. A. Sloane (njas(AT)research.att.com).