Finite Metacyclic Groups

January-10-11
10:43 AM

What's the "generic" finite metacyclic group?

$\mathbb{Z}_m \rtimes_\alpha \mathbb{Z}_n$ $\quad \alpha \in \mathbb{Z}$ s.t. $\alpha^m = 1 \bmod n$.

or $\quad n \mid \alpha^m - 1 = (\alpha - 1)(1 + \alpha + \ldots + \alpha^{m-1})$

start w/ any $m \& \alpha$, consider

$\mathbb{Z}_m \rtimes_\alpha \mathbb{Z}_{\alpha-1}$ – this is just $\mathbb{Z}_m \times \mathbb{Z}_{\alpha-1}$

$\mathbb{Z}_m \rtimes_\alpha \mathbb{Z}_{\frac{\alpha^m-1}{\alpha-1}}$: Eg. 1, $\mathbb{Z}_2 \rtimes_\alpha \mathbb{Z}_{\alpha+1}$ –
this is $D_{\alpha+1}$

Eg. 2, $\mathbb{Z}_3 \rtimes_\alpha \mathbb{Z}_{\alpha^2+\alpha+1}$