

```
In[1]:= f[n_] := n + Total[IntegerDigits[n]];
```

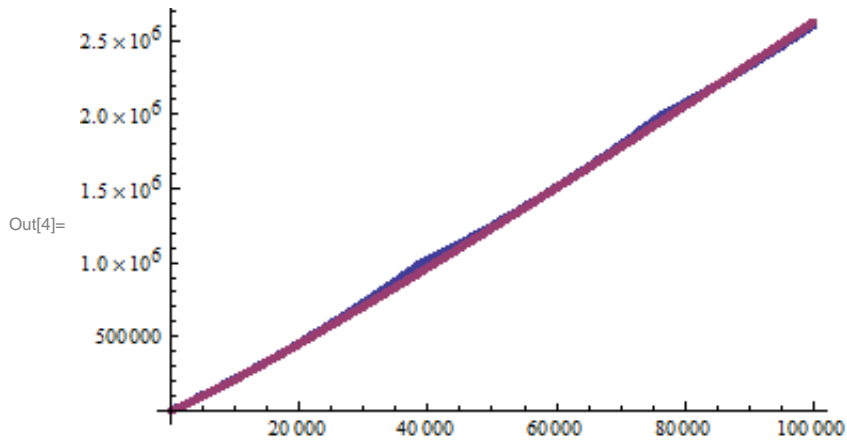
```
In[2]:= NestList[f, 1, 100]
```

```
Out[2]= {1, 2, 4, 8, 16, 23, 28, 38, 49, 62, 70, 77, 91, 101, 103, 107, 115, 122, 127, 137, 148,
161, 169, 185, 199, 218, 229, 242, 250, 257, 271, 281, 292, 305, 313, 320, 325, 335,
346, 359, 376, 392, 406, 416, 427, 440, 448, 464, 478, 497, 517, 530, 538, 554, 568,
587, 607, 620, 628, 644, 658, 677, 697, 719, 736, 752, 766, 785, 805, 818, 835, 851,
865, 884, 904, 917, 934, 950, 964, 983, 1003, 1007, 1015, 1022, 1027, 1037, 1048,
1061, 1069, 1085, 1099, 1118, 1129, 1142, 1150, 1157, 1171, 1181, 1192, 1205, 1213}
```

```
In[3]:= NestList[f, 44, 100]
```

```
Out[3]= {44, 52, 59, 73, 83, 94, 107, 115, 122, 127, 137, 148, 161, 169, 185, 199, 218, 229, 242, 250,
257, 271, 281, 292, 305, 313, 320, 325, 335, 346, 359, 376, 392, 406, 416, 427, 440, 448,
464, 478, 497, 517, 530, 538, 554, 568, 587, 607, 620, 628, 644, 658, 677, 697, 719, 736,
752, 766, 785, 805, 818, 835, 851, 865, 884, 904, 917, 934, 950, 964, 983, 1003, 1007,
1015, 1022, 1027, 1037, 1048, 1061, 1069, 1085, 1099, 1118, 1129, 1142, 1150, 1157,
1171, 1181, 1192, 1205, 1213, 1220, 1225, 1235, 1246, 1259, 1276, 1292, 1306, 1316}
```

```
In[4]:= ListPlot[{NestList[f, 1, 100 000], Table[5.25 n Log[10, n], {n, 100 000}]}] // Rasterize
```



```
In[5]:= N[NestList[f, 1, 1000 - 2] / Table[n Log[10, n], {n, 2, 1000}]]
```

```
Out[5]= {1.66096, 1.39727, 1.66096, 2.28908, 3.42693, 3.88797, 3.87558, 4.42468, 4.9, 5.41233,
5.40533, 5.31722, 5.67127, 5.72518, 5.34623, 5.1153, 5.08964, 5.02134, 4.88075, 4.93398,
5.01129, 5.14053, 5.10188, 5.2935, 5.40918, 5.64083, 5.65147, 5.70626, 5.6416, 5.55889,
5.62652, 5.60756, 5.60781, 5.64372, 5.5866, 5.51501, 5.4138, 5.39874, 5.3993, 5.42918,
5.5151, 5.58093, 5.61457, 5.59181, 5.58266, 5.59878, 5.55145, 5.60254, 5.62694, 5.707,
5.79387, 5.79954, 5.75098, 5.78772, 5.80193, 5.86502, 5.93476, 5.93413, 5.88626, 5.9134,
5.92108, 5.97221, 6.02965, 6.10153, 6.12874, 6.14645, 6.14715, 6.18692, 6.23273, 6.2234,
6.24402, 6.25631, 6.25347, 6.28602, 6.32425, 6.31283, 6.32863, 6.33703, 6.33181,
```

6.35887, 6.39127, 6.32206, 6.27941, 6.23168, 6.1731, 6.14562, 6.12455, 6.11542, 6.07794,
6.08618, 6.08296, 6.10699, 6.08711, 6.07823, 6.04315, 6.00363, 6.00082, 5.97769,
5.96, 5.95249, 5.92062, 5.88456, 5.83969, 5.8193, 5.80392, 5.79799, 5.81031, 5.81774,
5.81599, 5.79657, 5.78182, 5.77592, 5.74872, 5.75575, 5.7542, 5.77326, 5.79593, 5.78971,
5.76349, 5.76945, 5.76736, 5.78468, 5.80544, 5.79901, 5.77374, 5.77881, 5.7763,
5.79213, 5.81123, 5.83713, 5.84462, 5.84836, 5.84496, 5.85893, 5.87604, 5.86888,
5.87532, 5.87822, 5.87437, 5.88701, 5.90266, 5.89527, 5.90082, 5.90303, 5.89882, 5.91031,
5.92468, 5.91715, 5.92193, 5.92355, 5.9462, 5.95648, 5.96952, 5.93517, 5.913, 5.88822,
5.85801, 5.84249, 5.82999, 5.82326, 5.82777, 5.82939, 5.80066, 5.78597, 5.77417,
5.76788, 5.74826, 5.75014, 5.74667, 5.7563, 5.76837, 5.76215, 5.74315, 5.74476, 5.74125,
5.75031, 5.76172, 5.75558, 5.73717, 5.73855, 5.73501, 5.74357, 5.75437, 5.76978,
5.75179, 5.75283, 5.74915, 5.7571, 5.76723, 5.7818, 5.78475, 5.78537, 5.78145, 5.78874,
5.79814, 5.79191, 5.79452, 5.79489, 5.79088, 5.7977, 5.80655, 5.80033, 5.80264, 5.8028,
5.79872, 5.8051, 5.81346, 5.80726, 5.8093, 5.80926, 5.82339, 5.82928, 5.83708, 5.83081,
5.83252, 5.83221, 5.84562, 5.85105, 5.87581, 5.8694, 5.85344, 5.8529, 5.84855, 5.85367,
5.86059, 5.85433, 5.83881, 5.83824, 5.83397, 5.83889, 5.84555, 5.83944, 5.82435, 5.82375,
5.81957, 5.82429, 5.83071, 5.84058, 5.82581, 5.82511, 5.82093, 5.8254, 5.83153, 5.84099,
5.84185, 5.84099, 5.83675, 5.84091, 5.84669, 5.84077, 5.8415, 5.84056, 5.83634, 5.84029,
5.84581, 5.83997, 5.84059, 5.83958, 5.83539, 5.83914, 5.84443, 5.83865, 5.83917, 5.83811,
5.84789, 5.85141, 5.85641, 5.85064, 5.85101, 5.84984, 5.85923, 5.86252, 5.88074, 5.88835,
5.88846, 5.90038, 5.90924, 5.89896, 5.89021, 5.88443, 5.88448, 5.88307, 5.87879,
5.88167, 5.87314, 5.86749, 5.86753, 5.86615, 5.86196, 5.86477, 5.85644, 5.85092,
5.85095, 5.84959, 5.8455, 5.84824, 5.8523, 5.84687, 5.84684, 5.84546, 5.84142, 5.84404,
5.84796, 5.84259, 5.84252, 5.84113, 5.83712, 5.83964, 5.84342, 5.83813, 5.83801, 5.8366,
5.84412, 5.84649, 5.83871, 5.83349, 5.83333, 5.83191, 5.83922, 5.8415, 5.83384, 5.8287,
5.8285, 5.82707, 5.83419, 5.83637, 5.85066, 5.85637, 5.85603, 5.86526, 5.87204, 5.87399,
5.87711, 5.88256, 5.89268, 5.88041, 5.87641, 5.86776, 5.86033, 5.85525, 5.85482, 5.85323,
5.84935, 5.84092, 5.83367, 5.82874, 5.82835, 5.82683, 5.82307, 5.82492, 5.81783, 5.81299,
5.81262, 5.81113, 5.80745, 5.80927, 5.81216, 5.80739, 5.807, 5.80551, 5.80187, 5.80363,
5.80644, 5.80174, 5.80132, 5.79984, 5.80578, 5.80744, 5.80067, 5.79604, 5.7956, 5.79413,
5.79992, 5.80154, 5.79487, 5.79029, 5.78985, 5.78837, 5.79404, 5.7956, 5.80728, 5.81181,
5.81126, 5.81875, 5.82419, 5.82559, 5.82797, 5.83231, 5.84059, 5.83893, 5.84417, 5.84545,
5.84769, 5.85187, 5.8424, 5.83201, 5.82844, 5.82103, 5.81462, 5.81016, 5.80954, 5.80796,
5.81304, 5.80577, 5.79947, 5.7951, 5.79449, 5.79295, 5.79795, 5.79081, 5.78463, 5.78033,
5.77975, 5.77824, 5.78316, 5.77615, 5.77008, 5.76586, 5.7653, 5.76382, 5.76867, 5.76988,
5.7639, 5.75973, 5.75916, 5.75769, 5.76245, 5.76362, 5.76567, 5.76947, 5.76885, 5.77523,
5.77983, 5.78091, 5.78285, 5.78651, 5.7936, 5.79203, 5.79648, 5.79747, 5.79931, 5.80284,
5.80209, 5.80049, 5.80482, 5.81335, 5.82266, 5.82605, 5.83276, 5.8386, 5.85024, 5.84352,
5.83767, 5.83349, 5.83264, 5.83095, 5.83502, 5.82842, 5.82267, 5.81856, 5.81773, 5.81608,
5.8201, 5.81361, 5.80795, 5.80392, 5.80311, 5.8015, 5.80546, 5.80623, 5.80065, 5.79667,
5.79586, 5.79427, 5.79817, 5.79891, 5.80043, 5.8035, 5.80266, 5.80802, 5.81181, 5.81249,
5.81393, 5.8169, 5.82291, 5.82124, 5.82492, 5.82554, 5.8269, 5.82977, 5.82885, 5.82717,
5.83076, 5.83808, 5.8461, 5.84886, 5.85459, 5.85954, 5.86965, 5.87009, 5.87127, 5.87392,
5.87948, 5.87769, 5.87444, 5.87485, 5.87598, 5.87856, 5.88401, 5.88221, 5.87898,
5.87935, 5.88043, 5.88294, 5.88186, 5.88007, 5.88326, 5.8836, 5.88464, 5.88072, 5.87964,

5.87786, 5.88099, 5.88131, 5.88861, 5.89099, 5.88988, 5.89432, 5.89736, 5.89762,
5.89856, 5.90087, 5.90591, 5.90408, 5.90703, 5.90725, 5.90813, 5.91037, 5.90921,
5.91346, 5.91634, 5.92257, 5.92945, 5.93159, 5.93641, 5.94053, 5.94929, 5.95535,
5.96205, 5.97004, 5.96875, 5.9668, 5.96353, 5.96356, 5.96425, 5.96035, 5.95907, 5.95714,
5.95977, 5.95979, 5.96046, 5.95659, 5.95532, 5.95341, 5.95599, 5.956, 5.9624, 5.96431,
5.96302, 5.96681, 5.96932, 5.96929, 5.96988, 5.97173, 5.97609, 5.97415, 5.97659,
5.97653, 5.97708, 5.97888, 5.97756, 5.9812, 5.98359, 5.98906, 5.99511, 5.99684, 6.00101,
6.00454, 6.01235, 6.01767, 6.02359, 6.03069, 6.03473, 6.03815, 6.04578, 6.03467,
6.02421, 6.01498, 6.00819, 6.00083, 5.99229, 5.98677, 5.98186, 5.97815, 5.97149,
5.96959, 5.96651, 5.96639, 5.96156, 5.95792, 5.95136, 5.94424, 5.94123, 5.9359, 5.93116,
5.9276, 5.92116, 5.91416, 5.90603, 5.9008, 5.89617, 5.89271, 5.89154, 5.88981, 5.88694,
5.88179, 5.87724, 5.87383, 5.8676, 5.86591, 5.8631, 5.86312, 5.8637, 5.86034, 5.85418,
5.85253, 5.84976, 5.84979, 5.85037, 5.84706, 5.84098, 5.83935, 5.83662, 5.83666,
5.83725, 5.83893, 5.83786, 5.83624, 5.83354, 5.83357, 5.83414, 5.8309, 5.82984, 5.82824,
5.82557, 5.8256, 5.82617, 5.82296, 5.82192, 5.82034, 5.81769, 5.81772, 5.81829, 5.81512,
5.81408, 5.81252, 5.81467, 5.8147, 5.81525, 5.80736, 5.8016, 5.79534, 5.78804, 5.78339,
5.77927, 5.77621, 5.77524, 5.77375, 5.76656, 5.76197, 5.75792, 5.75492, 5.74934, 5.7479,
5.74544, 5.74554, 5.74616, 5.74319, 5.73768, 5.73626, 5.73383, 5.73394, 5.73456,
5.73163, 5.72618, 5.72479, 5.72239, 5.72251, 5.72313, 5.72475, 5.71935, 5.71797,
5.71559, 5.71571, 5.71633, 5.71793, 5.71705, 5.71568, 5.71332, 5.71343, 5.71403,
5.71119, 5.71032, 5.70896, 5.70662, 5.70673, 5.70733, 5.70451, 5.70365, 5.7023, 5.69999,
5.7001, 5.70069, 5.6979, 5.69705, 5.69572, 5.69774, 5.69785, 5.69843, 5.69566, 5.69481,
5.69348, 5.69548, 5.69558, 5.70041, 5.69766, 5.69255, 5.69123, 5.68897, 5.68907,
5.68963, 5.68691, 5.68185, 5.68055, 5.67832, 5.67842, 5.67899, 5.6763, 5.6713, 5.67002,
5.66781, 5.66792, 5.66849, 5.66997, 5.66502, 5.66375, 5.66156, 5.66167, 5.66223,
5.66371, 5.6629, 5.66163, 5.65946, 5.65956, 5.66011, 5.6575, 5.6567, 5.65544, 5.65329,
5.6534, 5.65394, 5.65135, 5.65056, 5.64932, 5.64719, 5.64729, 5.64783, 5.64526, 5.64448,
5.64325, 5.64511, 5.64521, 5.64574, 5.64319, 5.64241, 5.64118, 5.64303, 5.64312,
5.64757, 5.64896, 5.64816, 5.65085, 5.65266, 5.64882, 5.64542, 5.6429, 5.64211, 5.6409,
5.63882, 5.6389, 5.63554, 5.63304, 5.63227, 5.63107, 5.62901, 5.6291, 5.62577, 5.62331,
5.62254, 5.62136, 5.61933, 5.61942, 5.61993, 5.61748, 5.61673, 5.61555, 5.61354,
5.61363, 5.61413, 5.61171, 5.61096, 5.6098, 5.6078, 5.60789, 5.60839, 5.60599, 5.60525,
5.60409, 5.60583, 5.60591, 5.6027, 5.60032, 5.59958, 5.59844, 5.60016, 5.60025, 5.59706,
5.5947, 5.59397, 5.59284, 5.59455, 5.59463, 5.59876, 5.60005, 5.59931, 5.6018, 5.60348,
5.60354, 5.604, 5.60527, 5.60813, 5.60338, 5.60144, 5.5979, 5.59478, 5.59246, 5.59173,
5.5906, 5.58869, 5.5852, 5.58211, 5.57981, 5.5791, 5.57799, 5.5761, 5.57618, 5.57312,
5.57085, 5.57015, 5.56906, 5.56719, 5.56727, 5.56774, 5.56549, 5.56479, 5.56371,
5.56186, 5.56194, 5.56241, 5.56017, 5.55948, 5.55841, 5.56003, 5.5601, 5.55712, 5.5549,
5.55422, 5.55315, 5.55476, 5.55483, 5.55187, 5.54967, 5.549, 5.54794, 5.54953, 5.54961,
5.55346, 5.55466, 5.55397, 5.55629, 5.55786, 5.55792, 5.55835, 5.55953, 5.56219,
5.56113, 5.56267, 5.56271, 5.56313, 5.56429, 5.56026, 5.55586, 5.55407, 5.5508, 5.5479,
5.54575, 5.54508, 5.54404, 5.54556, 5.54232, 5.53945, 5.53732, 5.53666, 5.53563,
5.53715, 5.53394, 5.5311, 5.52899, 5.52833, 5.52731, 5.52883, 5.52564, 5.52283, 5.52074,
5.52009, 5.51908, 5.52059, 5.52066, 5.51787, 5.51579, 5.51515, 5.51415, 5.51565,
5.51572, 5.51614, 5.51727, 5.51663, 5.51881, 5.52029, 5.52035, 5.52076, 5.52187,

```
5.52438, 5.52338, 5.52483, 5.52488, 5.52527, 5.52637, 5.52571, 5.52471, 5.52614,
5.52931, 5.53282, 5.5339, 5.53635, 5.53845, 5.54297, 5.53987, 5.53713, 5.53508, 5.53441,
5.5334, 5.5348, 5.53173, 5.52901, 5.52698, 5.52632, 5.52532, 5.52671, 5.52366, 5.52097,
5.51895, 5.51831, 5.51732, 5.5187, 5.51873, 5.51605, 5.51405, 5.51341, 5.51243,
5.5138, 5.51383, 5.51419, 5.51522, 5.51458, 5.51661, 5.51797, 5.51798, 5.51833}
```

```
In[6]:= NestWhile[f, 1, (# < 12 345) &]
```

```
Out[6]= 12 358
```

```
In[7]:= $RecursionLimit = 123 456;
```

```
fa[m_, n_] := fa[m, n] = If[n > m, n, fa[m, f[n]]]
```

```
In[9]:= fa[12 345, 1]
```

```
Out[9]= 12 358
```

```
In[10]:= Union[fa[12 345, #] & /@ Range[10 000]]
```

```
Out[10]= {12 351, 12 357, 12 358}
```

```
In[11]:= fa[12 345, #] & /@ Range[100]
```

```
Out[11]= {12 358, 12 358, 12 351, 12 358, 12 358, 12 351, 12 358, 12 358, 12 357, 12 358, 12 358, 12 351,
12 358, 12 358, 12 351, 12 358, 12 358, 12 357, 12 358, 12 358, 12 351, 12 358, 12 358, 12 351, 12 358,
12 358, 12 357, 12 358, 12 358, 12 351, 12 358, 12 358, 12 351, 12 358, 12 358, 12 357,
12 358, 12 358, 12 351, 12 358, 12 358, 12 351, 12 358, 12 358, 12 357, 12 358, 12 358, 12 351, 12 358,
12 358, 12 351, 12 358, 12 358, 12 357, 12 358, 12 358, 12 351, 12 358, 12 358, 12 351, 12 358,
12 358, 12 358, 12 357, 12 358, 12 358, 12 351, 12 358, 12 358, 12 351, 12 358, 12 358,
12 357, 12 358, 12 358, 12 351, 12 358, 12 358, 12 351, 12 358, 12 358, 12 357, 12 358}
```

```
In[12]:= Clear[f1, fal];
```

```
f1[n_] := 22 + n + Total[IntegerDigits[n]];
```

```
fal[m_, n_] := fal[m, n] = If[n > m, n, fal[m, f1[n]]]
```

```
In[15]:= Union[fal[1 234 567, #] & /@ Range[1000]]
```

```
Out[15]= {1 234 594, 1 234 598, 1 234 604}
```