The Double of a Finite Group

October-07-10
7:53 AM

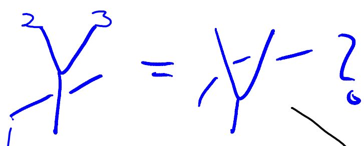
From Bakalov-Kirillov:

multiplication	$(\delta_g \otimes x)(\delta_h \otimes y) = \delta_{gx,xh}(\delta_g \otimes xy), \qquad x, y, g, h \in G, \longrightarrow x h^* = (xhx^{-1})^* x$
unit	$1 = \sum_{g\in G} \delta_g \otimes e,$
comultiplication	$\Delta(\delta_g \otimes x) = \sum_{g_1 g_2 = g} (\delta_{g_1} \otimes x) \otimes (\delta_{g_2} \otimes x), \qquad g, x \in G,$
counit	$\varepsilon(\delta_g \otimes x) = \delta_{g,e},$
antipode	$\gamma(\delta_g \otimes x) = \delta_{x^{-1}g^{-1}x} \otimes x^{-1}.$
R-matrix	$R = \sum_{g\in G} (\delta_g \otimes e) \otimes (1 \otimes g).$

Question. What, in the world of w-knots, is this?

Question. What's the natural integral?

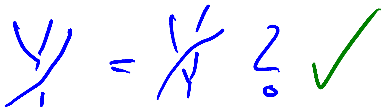
Question. What are "twisted doubles"?

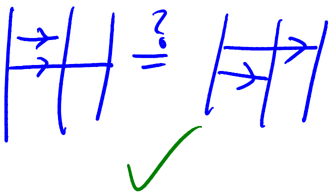


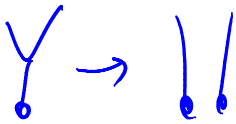
$\sum g\otimes\Delta(\check{g}) \qquad \sum_g\sum_h gh\otimes \check{g}\otimes\check{h}$ ✓

$\qquad\parallel \qquad\qquad\qquad \parallel$

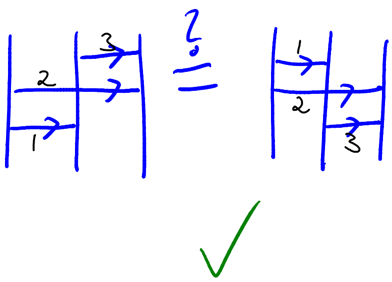
$\sum_{g,h} g\otimes g\check{h}^{-1}\otimes\check{h} = \sum gh\otimes\check{g}\otimes\check{h}$







well defined?



$\sum g_1^* g_2^* \otimes g_1 g_3^* \otimes g_2 g_3 \overset{?}{=} \sum g_2^* g_1^* \otimes g_3^* g_1 \otimes g_3 g_2$

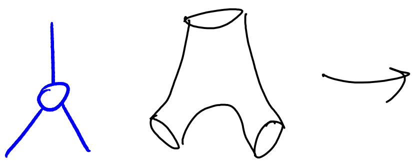
$\parallel \qquad\qquad\qquad \parallel$

$\sum g^* \otimes g g_3^* \otimes g g_3 \qquad \sum g^* \otimes g_3^* g \otimes g_3 g$

$\parallel \qquad\qquad\qquad \parallel$

$\sum g^* \otimes (g g_3 g^{-1})^* g \otimes g g_3 = \sum g^* \otimes h^* g \otimes hg$

Is there a smooth vertex

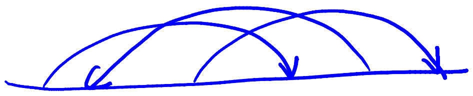


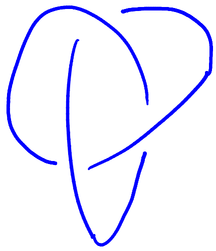
Take G as group like

Take G^* as?

It's not an operation, it's a $D(G)^{\otimes 3}$ module!

$$S = D(G)^{\otimes 3} \Big/ \begin{matrix} g\otimes g\otimes g = 0 \\ \check{g}\otimes 1\otimes 1 = 1\otimes \check{g}\otimes 1 = 1\otimes 1\otimes \check{g} \end{matrix} \quad ?$$

 $\longrightarrow$?

 $\longrightarrow$?

What's a pro-finite group?

Are knot groups pro-finite?

Are there two knots with the same pro-finite comp.?

Conjecture. $Z(K) \sim \sum_{\rho \in \mathrm{Hom}(\pi_1(K), G)} \rho(m)^* \cdot \rho(l)$ where m & l are the meridian & longitude.

[The good proof should involve circuit algebras].

Facts on $\pi_1(K)$: 1. l & m always commute.

2. l is a product of conjugates of m.

Co-invariants. What's $D(G)/[D(G), D(G)]$ (in the vector-space sense)?

$$g^* x h^* y = g^* (h^{x^{-1}})^* xy = \delta_{g, h^{x^{-1}}} g^* xy = \#1$$

$$h^* y g^* x = h^* (g^{y^{-1}})^* yx = \delta_{h, g^{y^{-1}}} h^* yx = \#2$$

$$D(G)_{D(G)} \stackrel{?}{=} G\times G / \text{conj by } G. \Big\} \text{ naive.}$$

$$\#1 \longrightarrow \begin{cases} (g, xy) & \text{if } g = xhx^{-1} \Leftrightarrow g^x = h \\ 0 & \text{otherwise} \end{cases}$$

$$\#2 \longrightarrow \begin{cases} (ygy^{-1}, yx) & \text{if } h = ygy^{-1} \Leftrightarrow h^y = g \\ 0 & \text{otherwise} \end{cases}$$

$$D(G)_{D(G)} \sim \{(g, x) : [g, x] = 1\} / \text{conj} ? \quad ✓$$

$$D(G)/_{D(G)} \sim \{(g, x) : [g,x]=1\} / \text{conj}?$$ ✓ (tentative)

$$g^{xy}=g \leftrightarrow h^y=g$$