The Alexander Quandle

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From Joyce:

Theorem 17.3. *Let the Alexander invariant A be given the quandle structure*

$$x \triangleright y = t(x-y)+y, \qquad x \triangleright^{-1} y = t^{-1}(x-y)+y.$$

Then with this structure, A is isomorphic to AbQ.

$$x \wedge y = tx + (1-t)y$$

$$(x \wedge y) \wedge z = t^2 x + (t-t^2)y + (1-t)z$$

$$(tx+(1-t)z) \wedge (ty+(1-t)z) = t^2 x + (t-t^2)z + (t-t^2)y + (1-t)^2 z$$

$$= t^2 x + (t-t^2)y + (1-t)z$$

$$(tx+b)^{-1} \circ (tx+a) \circ (tx+b) =$$

$$\begin{pmatrix} t & b \\ 0 & 1 \end{pmatrix}^{-1} \begin{pmatrix} t & a \\ 0 & 1 \end{pmatrix} \begin{pmatrix} t & b \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} t^{-1} & -b/t \\ 0 & 1 \end{pmatrix} \begin{pmatrix} t^2 & t(a+b) \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} t & a+b-t^{-1}b \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} t & b \\ 0 & 1 \end{pmatrix} \begin{pmatrix} t & a \\ 0 & 1 \end{pmatrix} \begin{pmatrix} t^{-1} & -b/t \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} t^2 & t(a+b) \\ 0 & 1 \end{pmatrix} \begin{pmatrix} t^{-1} & -b/t \\ 0 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} t & -bt + t(a+b) \\ 0 & 1 \end{pmatrix} =$$

Idiotic