Kamnitzer: Modular tensor categories and modular functors:
braided tensor categories and quantum groups

September-22-10
9:08 AM

Following Bakalov–Kirillov

http://www.math.sunysb.edu/~kirillov/tensor/tensor.html

G: Finite group, V a representation over $\mathbb{C}$: $\rho: G \to GL(V)$.

<u>Operations</u> 1. Direct sum. (additive category)

Also an Abelian category – has kernels & cokers.

<u>Aside</u> An Abelian category is the category of modules over some ring. For $Rep(G)$, this is the group ring.

<u>Definition</u> A simple object in an Abelian category is one with no non-trivial subobject.

The category is "semi-simple" if every object is a finite direct sum of simple objects.

<u>Claim</u> $Rep(G)$ is semi-simple.

<u>More operations</u> 2. Tensor products.

3. $V^* = Hom(V, \mathbb{C})$ w/ $(g\alpha)(v) = \alpha(g^{-1}v)$ $\quad \alpha \in V^*$, $v \in V$

$\alpha_{U,V,W} : (U \otimes V) \otimes W \to U \otimes (V \otimes W)$, satisfies the pentagon.... Such a category, which also has a unit (the trivial rep for $Rep(G)$) is called a "monoidal category".

Likewise there is $\sigma: U \otimes V \to V \otimes U$, which along with α should satisfy hexagons.

In $Rep(G)$, we also have $\sigma^2 = I$.

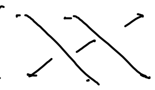
Definition A braided monoidal category, a symmetric monoidal category.

Suppose $\mathcal{C}$ is a semi-simple monoidal category, $K(\mathcal{C})$ the free Abelian group generated by the simple objects of $\mathcal{C}$. $\Lambda := \{\text{set of simple objects}\}$

$$V_\lambda \otimes V_\mu = \bigoplus_\nu N_{\lambda\mu}^{\nu} V_\nu$$

This defines a product on $K(\mathcal{C})$, so $K(\mathcal{C})$ is an associative ring, "the fusion ring", "the Grothendieck ring". If $\mathcal{C}$ is braided monoidal, $K(\mathcal{C})$ is commutative.

Let B_n be the braid group:

1. $B_n = \langle \sigma_1 \ldots \sigma_{n-1} : \ldots\ldots \rangle$
2. Pictures: $\{$  $\}$.
3. $\pi_1(\mathbb{C}^n \setminus \{z_i = z_j\} / S_n)$.

$PB_n = \ker(B_n \to S_n)$

Given a braided monoidal category, objects $V_1 \ldots V_n$, and a parenthetization, get a representation of PB_n.