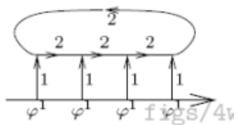
From http://www.math.toronto.edu/~drorbn/papers/WKO/:

3.6.3. *Example: The 2 Dimensional Non-Abelian Lie Algebra.* Let $\mathfrak{g}$ be the Lie algebra with two generators $x_{1,2}$ satisfying $[x_1, x_2] = x_2$, so that the only non-vanishing structure constants b_{ij}^k of $\mathfrak{g}$ are $b_{12}^2 = -b_{21}^2 = 1$. Let $\varphi^i \in \mathfrak{g}^*$ be the dual basis of x_i; by an easy calculation, we find that in $I\mathfrak{g}$ the element φ^1 is central, while $[x_1, \varphi^2] = -\varphi^2$ and $[x_2, \varphi^2] = \varphi^1$. We calculate $\mathcal{T}_{\mathfrak{g}}^w(D_L)$, $\mathcal{T}_{\mathfrak{g}}^w(D_R)$ and $\mathcal{T}_{\mathfrak{g}}^w(w_k)$ using the "in basis" technique of Equation (18). The outputs of these calculations lie in $\mathcal{U}(I\mathfrak{g})$; we display these results in a PBW basis in which the elements of $\mathfrak{g}^*$ precede the elements of $\mathfrak{g}$:

$$\begin{aligned}
\mathcal{T}_{\mathfrak{g}}^w(D_L) &= x_1\varphi^1 + x_2\varphi^2 = \varphi^1 x_1 + \varphi^2 x_2 + [x_2, \varphi^2] = \varphi^1 x_1 + \varphi^2 x_2 + \varphi_1, \\
\mathcal{T}_{\mathfrak{g}}^w(D_R) &= \varphi^1 x_1 + \varphi^2 x_2, \\
\mathcal{T}_{\mathfrak{g}}^w(w_k) &= (\varphi^1)^k.
\end{aligned} \tag{19}$$

For the last assertion above, note that all non-vanishing structure constants b_{ij}^k in our case have $k = 2$, and therefore all indices corresponding to edges that exit an internal vertex must be set equal to 2. This forces the "hub" of w_k to be marked 2 and therefore the legs to be marked 1, and therefore w_k is mapped to $(\varphi^1)^k$.



$[\downarrow,\rightarrow]$	$x_1^{n_1}$	$x_2^{n_2}$	$\varphi_1^{p_1}$	$\varphi_2^{p_2}$
x_1	0	$n_2 x_2^{n_2}$	0	$-p_2\varphi_2^{p_2}$
x_2	$x_2\sum_{j=0}^{n-1}\binom{n}{j}x_1^j$	0	0	$p_2\varphi_1\varphi_2^{p_2-1}$
φ_1	0	0	0	0
φ_2	$\varphi_2\sum_{j=0}^{n-1}\binom{n}{j}x_1^j$	$-n_2\varphi_1\varphi_2^{n_2-1}$	0	0

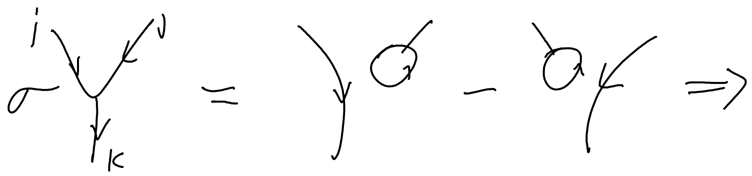
$$e^{\alpha x_1}x_2e^{-\alpha x_1} = e^{\alpha\, \mathrm{ad}\, x_1}x_2 = e^{\alpha}x_2$$

$$\Rightarrow \quad e^{\alpha x_1}x_2 = e^{\alpha}x_2e^{\alpha x_1} = x_2e^{\alpha x_1} + (e^{\alpha}-1)x_2e^{\alpha x_1}$$

$$\Rightarrow \quad [x_2, e^{\alpha x_1}] = (1-e^{\alpha})x_2e^{\alpha x_1} \qquad \left(\text{of course, } [x_2, e^{\alpha x_2}]=0\right)$$

$$\Rightarrow \quad [x_2, x_1^n] = x_2\left(x_1^n - \sum_{j=0}^{n}\binom{n}{j}x_1^j\right) = -x_2\sum_{j=0}^{n-1}\binom{n}{j}x_1^j$$

The Archibald relation: $\sigma\in\{\pm 1\}$



$$\sigma b_{ij}^k \stackrel{?}{=} \delta_k^i\delta_j^1 - \delta_i^1\delta_k^j$$

If $k=1$, $0=0$

if $k=2$, $i=1$, $j=2$:

So $\sigma = -1$! $\sigma = 0 - 1$

On to the Alexander polynomial —