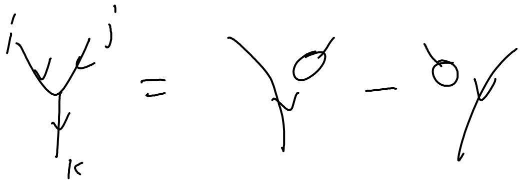
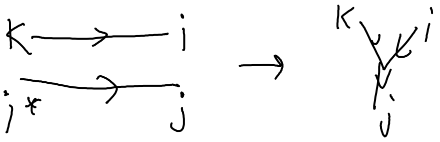
Link Relations in the w-Alexander Envelope

August-18-10
8:29 AM



$$[a_{ik}, a_{jk}] = x_j a_{ik} - x_i a_{jk}$$

(j^*j)-relations:

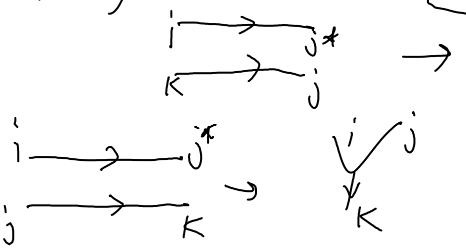


$$0 = \sum_k \frac{\partial Z}{\partial a_{ki}} \cdot (x_i a_{kj} - x_k a_{ij}) + s_0 \delta_{ij} x_i$$

LR1

Same signs by 2

(jj^*)-relations:



$$0 = s_1 \delta_{ij} x_i + \sum_k \frac{\partial Z}{\partial a_{kj}} (x_i a_{kj} - x_k a_{ij}) + \sum_k \frac{\partial Z}{\partial a_{jk}} (x_j a_{ik} - x_i a_{jk})$$

LR2

1 The relative sign of these two is forced.

Signs to be fixed later....

Added August 24: Fixing the bloody signs.

Principles: 1. $i \longrightarrow j^*$ acting on $j \longrightarrow j$ must give 0. Fixes 1.

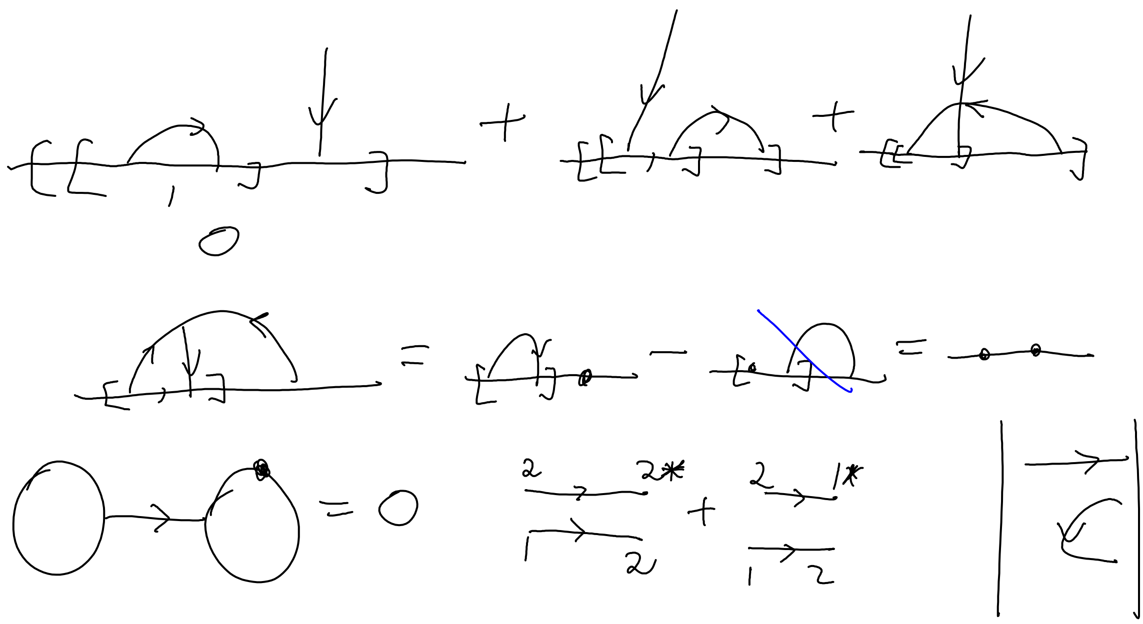
2. $(ij^*)+(ik^*)$ acting on (jk) must give 0. Fixes 1.

(This is $\overrightarrow{4T}$)

3. Probably, (jj^*) and (j^*j) act in the same manner (or opposite) on a_{ij}. Fixes 2.

Confirm! ..... Seems ok.

Aside: $\longleftrightarrow$ is invariant under everything



~~Conjecture~~ Theorem 1 (based on wAlexanderLinkRelations.nb) No matter the signs, the quotient is precisely all the x-free generators.

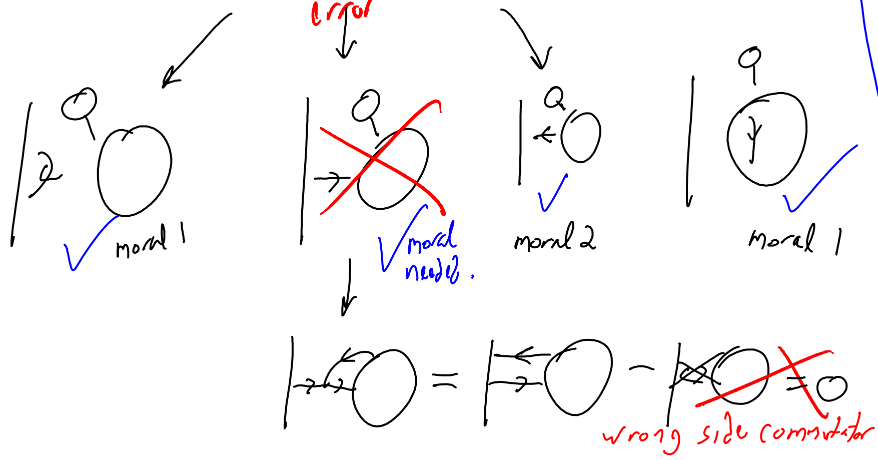
In the proof, only (ij^*) relations are needed!
(Proof only works with the "correct" signs)

I need to have a plausable picture explaining how the MVA is a f.t. invariant of closed classical links yet $A^w(\bigcirc n)$ has room only for linking links yet $A^w(\bigcirc n)$ has room only for linking numbers. This must involve cutting open one component, like in the knot case, but how? And how can I integrate/explain the fact that the end result is independent of the cut component?

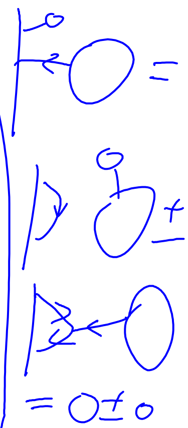
Link relations in $\mathcal{A}(|\bigcirc)$:

a[1, 1] x[2], a[1, 2] x[2], a[2, 1] x[2], a[2, 2] x[2], x[1] x[2], x[2]^2

error



What about a[2,1]x[1]?



<u>General morals</u>

1. If D has a round-only component that has a blob, then D=0.
2. If a round component has all tails on it, and a blob, then D=0.

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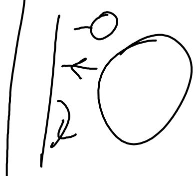
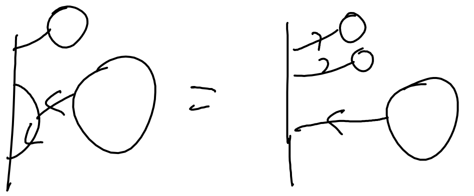
a[1, 1]^2 a[2, 1] x[1]
is a link relation
in $A(\uparrow\bigcirc)$

why?

So is
a[1, 1] a[2, 1] x[1]

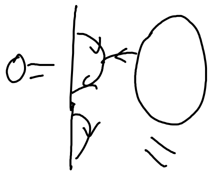


why? ✓



$= a[2,1]\, x[1]^2$

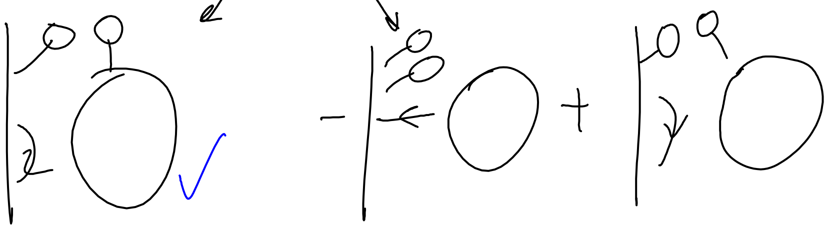
$0 =$

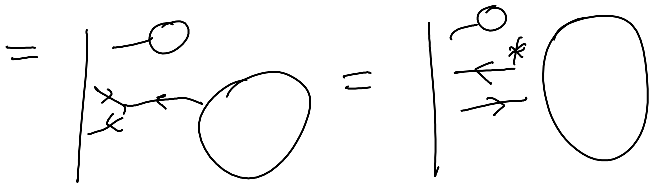


$abc - bac$
$= abc - bca$
$= abc - acb = 0$

wanted − ○

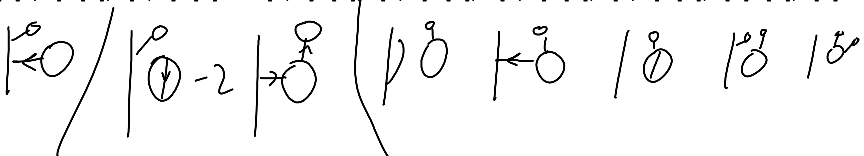
a[2,1]x[1]^2 is a relation too, and according to Mathematica it comes from the raw relations in {a[1, 1] x[1] x[2], -a[2, 1] x[1]^2 + a[1, 1] x[1] x[2]}



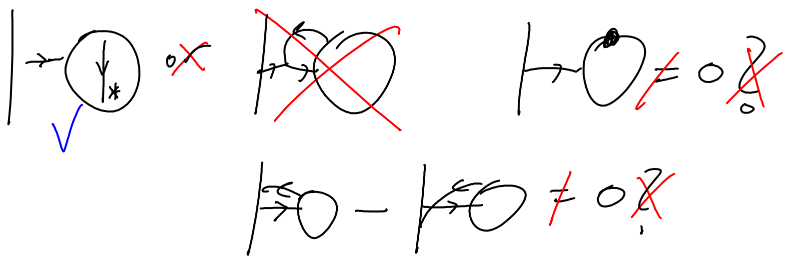


checks, but I still need to draw a moral.

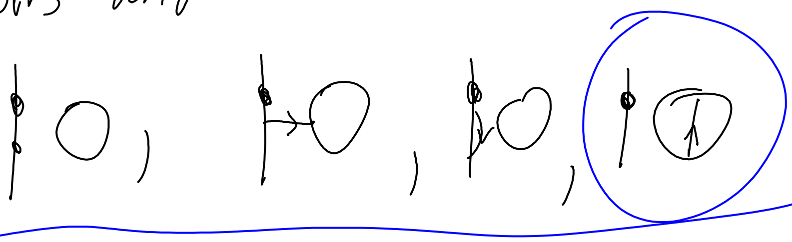
Further degree 2 relations: {a[2, 1] x[1], a[2, 2] x[1] - 2 a[1, 2] x[2], a[1, 1] x[2], a[2, 1] x[2], a[2, 2] x[2], x[1] x[2], x[2]^2}



comes from

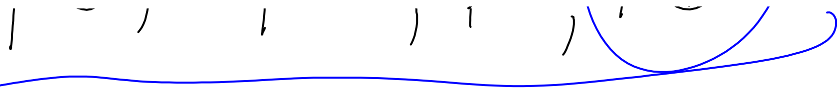


So the quotient is generated by the $\binom{4+1}{2}=10$ linking numbers and

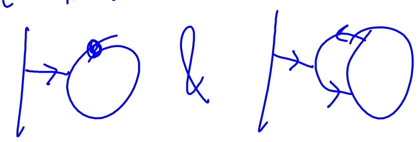


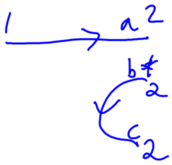
Eliminated if a sign is flipped.

... comrades: ... ?



Why? Comrades:





$1 \to a^2$

$b^*_2 \to (\pm a_{12} x_2) + a_{12} x_2 - a_{22} x_1$

c_2

top: $[b, ac+ca] = bac + bca - acb - cab$

bottom: $[c, ab+ba] = cab + cba - abc - bac$

$\to a, b_2, c^*_2 \to$ same up to signs.

top+bottom $= bca - acb + cba - abc = [bc+cb, a] \underset{4T}{=} 0$

$\Rightarrow$ So the two rels must be the same, w/ opposite signs

{s0, s1} = {1, 1}; Study[8]

{2,1}->{6,1,5}
{2,2}->{21,7,14}
{2,3}->{56,26,30}
{2,4}->{126,71,55}
{2,5}->{252,161,91}
{2,6}->{462,322,140}
{2,7}->{792,588,204}
{2,8}->{1287,1002,285}

$\dim g_m A^{ax+b}(\uparrow \bigcirc)$ perhaps

{{{6, 1, 5}, {21, 7, 14}, {56, 26, 30}, {126, 71, 55}, {252, 161, 91}, {462, 322, 140}, {792, 588, 204}, {1287, 1002, 285}},
{5, 14, 30, 55, 91, 140, 204, 285}, 1/6 (1 + m) (2 + m) (3 + 2 m)}

{s0, s1} = {-1, -1}; Study[8]

{2,1}->{6,1,5}
{2,2}->{21,7,14}
{2,3}->{56,24,32}
{2,4}->{126,63,63}
{2,5}->{252,141,111}
{2,6}->{462,282,180}
{2,7}->{792,518,274}
{2,8}->{1287,890,397}

$\dim g_m A^{pA}(\uparrow \bigcirc)$ perhaps

{{{6, 1, 5}, {21, 7, 14}, {56, 24, 32}, {126, 63, 63}, {252, 141, 111}, {462, 282, 180}, {792, 518, 274}, {1287, 890, 397}},
{5, 14, 32, 63, 111, 180, 274, 397}, 1/6 (6 + 17 m + 3 m^2 + 4 m^3)}