

Do All u-Alexander Relations Follow from w-Alexander Relations?

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The w-Alexander relation is $\begin{matrix} \nearrow \\ \searrow \end{matrix} = \begin{matrix} \searrow \\ \nearrow \end{matrix} - \begin{matrix} \nearrow \\ \searrow \end{matrix}$.

Does it imply all the relations in Jana's Thesis?

Relations on the Level of Chord Diagrams		
Smoothing Our proof p. 72	$\begin{matrix} \uparrow \\ \uparrow \end{matrix} = x \begin{matrix} \uparrow \\ \downarrow \end{matrix}$	Only if both arcs are labeled with x [FOKV97].
Our proof p. 72	$\begin{matrix} \uparrow \\ \downarrow \end{matrix} - \begin{matrix} \downarrow \\ \uparrow \end{matrix} = xy \begin{matrix} \uparrow \\ \uparrow \end{matrix}$	This is a consequence of Conway's second identity for knots and links [Mur99].
No Deep Vertices Our proof p. 73	$\begin{matrix} \circ \\ \downarrow \end{matrix} = 0$	✓
Blob Cutting	$\begin{matrix} \circ \\ \downarrow \end{matrix} = 0$	✓
Half Blobs	$\begin{matrix} \circ \\ \downarrow \end{matrix} := \begin{matrix} \circ \\ \downarrow \end{matrix}$	✓
The H Relation Our proof p. 65	$\begin{matrix} \circ \\ \downarrow \end{matrix} = \begin{matrix} \circ \\ \downarrow \end{matrix} - \begin{matrix} \circ \\ \downarrow \end{matrix} + \begin{matrix} \circ \\ \downarrow \end{matrix} - \begin{matrix} \circ \\ \downarrow \end{matrix}$	This is a direct consequence of the second relation. On any chord diagram there must be an even number of 'half blobs'.
4Y Our proof p. 74	$\begin{matrix} \circ \\ \downarrow \end{matrix} - \begin{matrix} \circ \\ \downarrow \end{matrix} + \begin{matrix} \circ \\ \downarrow \end{matrix} - \begin{matrix} \circ \\ \downarrow \end{matrix} = 0$	✓ ①
H. Murakami's Second Relation Our proof p. 75		
$4x_j \left(\begin{matrix} \uparrow \\ \uparrow \\ \uparrow \\ \uparrow \end{matrix} - \begin{matrix} \uparrow \\ \uparrow \\ \uparrow \\ \uparrow \end{matrix} \right) + 2(x_i - x_k) \left(\begin{matrix} \uparrow \\ \uparrow \\ \uparrow \\ \uparrow \end{matrix} + \begin{matrix} \uparrow \\ \uparrow \\ \uparrow \\ \uparrow \end{matrix} \right) + (x_k - x_i)(x_i x_k + x_j^2) \left(\begin{matrix} \uparrow \\ \uparrow \\ \uparrow \\ \uparrow \end{matrix} \right) = 0$		
H. Murakami's Third Relation Our proof p. 75	$\begin{matrix} \uparrow \\ \uparrow \end{matrix} = x_i \begin{matrix} \uparrow \\ \uparrow \end{matrix}$?
H. Murakami's Fourth Relation	$\begin{matrix} \uparrow \\ \uparrow \end{matrix} = x_i^{-1} \begin{matrix} \uparrow \\ \uparrow \end{matrix}$?
H. Murakami's Fifth Relation	$\left(\begin{matrix} \uparrow \\ \uparrow \end{matrix} \right) = 0$	

<http://www.math.toronto.edu/~ifa/>

Thesis page 56.

Might be new! Wrong as stated here! See posts below.
Satisfied by linking numbers.

$$Y = \begin{matrix} \nearrow \\ \searrow \end{matrix} + \begin{matrix} \searrow \\ \nearrow \end{matrix} + \begin{matrix} \nearrow \\ \searrow \end{matrix} = \begin{matrix} \searrow \\ \nearrow \end{matrix} - \begin{matrix} \nearrow \\ \searrow \end{matrix} + \dots$$

so $\begin{matrix} \uparrow \\ \uparrow \end{matrix} = \begin{matrix} \uparrow \\ \uparrow \end{matrix}$

①: $0 = \begin{matrix} \uparrow \\ \uparrow \end{matrix} = \begin{matrix} \uparrow \\ \uparrow \end{matrix} = 4V$

①:

$$0 = \text{[diagram 1]} = \text{[diagram 2]} = 4Y$$

Mastered from Jan's Thesis, page 65:

$$\text{[diagram 1]} = \text{[diagram 2]} - \text{[diagram 3]} + \text{[diagram 4]} - \text{[diagram 5]}$$

Added Sep 18, 2014
sign here may still be wrong.