Accounting for Env(pA) Weight-Systems

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2:47 PM

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In[65]:= Print[# → pW[2, wJD[#]]] & /@ Gens[2, 2]; RRels[2, 2]
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(done @ $S_0 = S_1 = -1$)

$a[1,1]^2 \to 0$
$a[1,1]\,a[1,2] \to -3\,x[1]^2$
$a[1,2]^2 \to -4\,x[1]^2$
$a[1,1]\,a[2,1] \to 0$
$a[1,2]\,a[2,1] \to -2\,x[1]\,x[2]$
$a[2,1]^2 \to 0$
$a[1,1]\,a[2,2] \to 0$
$a[1,2]\,a[2,2] \to -3\,x[1]\,x[2]$
$a[2,1]\,a[2,2] \to 0$
$a[2,2]^2 \to 0$

Linking numbers

✓ $a[1,1]\,x[1] \to 0$ $SVA_1 \cdot lk_{11}$
✓ $a[1,2]\,x[1] \to -2\,x[1]^2$
✓ $a[2,1]\,x[1] \to 0$ Rel
✓ $a[2,2]\,x[1] \to 0$ Rel
✓ $x[1]^2 \to 0$ SVA_1
✓ $a[1,1]\,x[2] \to 0$ Rel
✓ $a[1,2]\,x[2] \to -2\,x[1]\,x[2]$
✓ $a[2,1]\,x[2] \to 0$ Rel
✓ $a[2,2]\,x[2] \to 0$ Rel
✓ $x[1]\,x[2] \to 0$ Rel
✓ $x[2]^2 \to 0$ Rel

pA

```
Out[65]= {a[2, 1] x[1], a[2, 2] x[1], a[1, 1] x[2], a[2, 1] x[2], a[2, 2] x[2], x[1] x[2], x[2]^2}
```

Warning: $W_1, W_2 \in Env(pA) \not\Rightarrow W_1 \cdot W_2 \in Env(pA)$

Example $SVA_1 \cdot lk_{22} \notin Env(pA)$ as it is $\neq 0$ on ↗

and on ↖ $= 0$

Same in degree 3, but excluding linking numbers:

✓ $a[1,1]^2\,x[1] \to 0$
$a[1,1]\,a[1,2]\,x[1] \to -12\,x[1]^3$ $1^4 2$ ✓
$a[1,2]^2\,x[1] \to -12\,x[1]^3$ $1^3 2^2$ ✓
✓ $a[1,1]\,a[2,1]\,x[1] \to 0$ rel
✓ $a[1,2]\,a[2,1]\,x[1] \to -6\,x[1]^2\,x[2]$ rel w/ $a_{11}a_{12}x_2$ $1^3 2^2$
✓ $a[2,1]^2\,x[1] \to 0$ rel
✓ $a[1,1]\,a[2,2]\,x[1] \to 0$ rel
✓ $a[1,2]\,a[2,2]\,x[1] \to -6\,x[1]^2\,x[2]$ rel w/ $a_{12}^2 x_2$ $1^2 2^3$
✓ $a[2,1]\,a[2,2]\,x[1] \to 0$ rel
✓ $a[2,2]^2\,x[1] \to 0$ rel
✓ $a[1,1]\,x[1]^2 \to 0$ $SVA_1 \cdot lk_1$
$a[1,2]\,x[1]^2 \to -6\,x[1]^3$ $1^3 2$ ✓
✓ $a[2,1]\,x[1]^2 \to 0$ rel

$a[1,1]^2 x[1] \to 0$

$a[1,1]\,a[1,2]\,x[1] \to -12\,x[1]^3$ $1^4 2$

$a[1,2]^2 x[1] \to -12\,x[1]^3$ $1^3 2^2$

$a[1,1]\,a[2,1]\,x[1] \to 0$ rel

$a[1,2]\,a[2,1]\,x[1] \to -6\,x[1]^2\,x[2]$ rel w/ $a_{11}a_{12}x_2$ $1^3 2^2$

$a[2,1]^2 x[1] \to 0$ rel

$a[1,1]\,a[2,2]\,x[1] \to 0$ rel

$a[1,2]\,a[2,2]\,x[1] \to -6\,x[1]^2\,x[2]$ rel w/ $a_{12}^2 x_2$ $1^2 2^3$

$a[2,1]\,a[2,2]\,x[1] \to 0$ rel

$a[2,2]^2 x[1] \to 0$ rel

$a[1,1]\,x[1]^2 \to 0$ SVA, lk,

$a[1,2]\,x[1]^2 \to -6\,x[1]^3$ $1^3 2$

$a[2,1]\,x[1]^2 \to 0$ rel

$a[2,2]\,x[1]^2 \to 0$ rel

$x[1]^3 \to 0$ SVA,

$a[1,1]^2 x[2] \to 0$ rel

$a[1,1]\,a[1,2]\,x[2] \to -6\,x[1]^2\,x[2]$ $1^3 2^2$

$a[1,2]^2 x[2] \to -12\,x[1]^2\,x[2]$ $1^2 2^3$

$a[1,1]\,a[2,1]\,x[2] \to 0$ rel

$a[1,2]\,a[2,1]\,x[2] \to -6\,x[1]\,x[2]^2$ $1^2 2^3$

$a[2,1]^2 x[2] \to 0$ rel

$a[1,1]\,a[2,2]\,x[2] \to 0$ rel

$a[1,2]\,a[2,2]\,x[2] \to -12\,x[1]\,x[2]^2$ 12^4

$a[2,1]\,a[2,2]\,x[2] \to 0$ rel

$a[2,2]^2 x[2] \to 0$ rel

$a[1,1]\,x[1]\,x[2] \to 0$ rel

$a[1,2]\,x[1]\,x[2] \to -4\,x[1]^2\,x[2]$ $1^2 2^2$

$a[2,1]\,x[1]\,x[2] \to 0$ rel

$a[2,2]\,x[1]\,x[2] \to 0$ rel

$x[1]^2 x[2] \to 0$ rel

$a[1,1]\,x[2]^2 \to 0$ rel

$a[1,2]\,x[2]^2 \to -6\,x[1]\,x[2]^2$ 12^3

$a[2,1]\,x[2]^2 \to 0$ rel

$a[2,2]\,x[2]^2 \to 0$ rel

$x[1]\,x[2]^2 \to 0$ rel

$x[2]^3 \to 0$ rel

I can only explain 3 of the highlighted diagrams using pA. For the rest, use the grading by the "per-strand leg count."

$$\{a[1,1]\,a[2,1]\,x[1],\ a[1,2]\,a[2,1]\,x[1],\ a[1,1]\,a[1,2]\,x[2],$$
$$a[2,1]^2 x[1],\ a[1,1]\,a[2,2]\,x[1],\ a[1,2]\,a[2,2]\,x[1] - \frac{1}{2}\,a[1,2]^2\,x[2],$$
$$a[2,1]\,a[2,2]\,x[1],\ a[2,2]^2 x[1],\ a[2,1]\,x[1]^2,\ a[2,2]\,x[1]^2,\ a[1,1]^2\,x[2],$$
$$a[1,1]\,a[2,1]\,x[2],\ a[2,1]^2 x[2],\ a[1,1]\,a[2,2]\,x[2],\ a[2,1]\,a[2,2]\,x[2],$$
$$a[2,2]^2 x[2],\ a[1,1]\,x[1]\,x[2],\ a[2,1]\,x[1]\,x[2],\ a[2,2]\,x[1]\,x[2],$$
$$x[1]^2 x[2],\ a[1,1]\,x[2]^2,\ a[2,1]\,x[2]^2,\ a[2,2]\,x[2]^2,\ x[1]\,x[2]^2,\ x[2]^3\}$$