Pensieve header: Debugging the "SubLink" program, with Iva Halacheva.

```
<< KnotTheory`
```

```
Loading KnotTheory` version of August 22, 2010, 13:36:57.55.
Read more at http://katlas.org/wiki/KnotTheory.
```

```
SubLink::usage = "SubLink[pd, js] returns the
    sublink of pd made out of the components of pd in the list js.";
SubLink[pd_PD, js_List] := Module[
   {k, t0, t, t1, t2, s0, s1},
   s0 = Skeleton[pd];
   (* t0 contains the list of edges that should appear in the sublink *)
   t0 = Flatten[List @@@ s0[[js]]];
   (* t is pd with all edges not in t0 removed;
   this means that some crossings will now involve just 0 or 2 edges. *)
   t = pd /. x_X :> Select[x, MemberQ[t0, #] &];
   (* Remove all "empty" crossings from t: *)
   t = DeleteCases[t, X[] | Loop[]];
   (* Remove all "valency 2" crossings from t,
   while also removing not-longer-necessary edge labels: *)
   k = 1;
   While[
    k ≤ Length[t],
    If[Length[t[[k]]] == 2,
      t = Delete[t, k] /. (Rule @@ t[[k]]),
      (* else *) ++k
     ];
   ];
   (* We have to manually "re-add" all skeleton components that "disappeared": *)
   s1 = Union[Flatten[List @@ List @@@ t]];
   Do[
    If[
     MemberQ[js, k] && (And @@ (FreeQ[s1, #] & /@ s0[[k]])),
     AppendTo[t, Loop[s0[[k, 1]]]];
     AppendTo[s1, s0[[k, 1]]]
    ],
    {k, Length[s0]}
   ];
   (* t1 will have all edge-labels still appearing in t;
   it is used to relabel t so that the edge labels will be consecutive *)
   t1 = Sort[s1];
   t2 = Thread[(t1) → Range[Length[t1]]];
   t /. t2
  ];
SubLink[pd_PD, j_] := SubLink[pd, {j}];
SubLink[L_, js_] := SubLink[PD[L], js];
```

```
? SubLink
```

SubLink[pd, js] returns the sublink of pd made out of the components of pd in the list js.

```
Ls = Link /@ {"L7a7", "L10a149"}
```

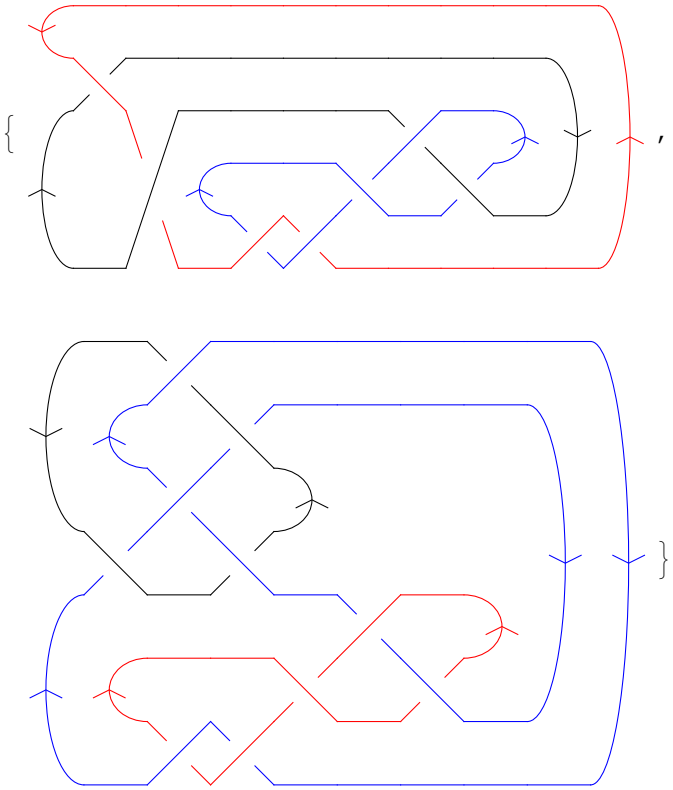
```
{Link[7, Alternating, 7], Link[10, Alternating, 149]}
```

```
DrawMorseLink /@ Ls
```

KnotTheory::loading : Loading precomputed data in PD4Links`.

KnotTheory::credits : MorseLink was added to KnotTheory` by Siddarth Sankaran
at the University of Toronto in the summer of 2005.

KnotTheory::credits : DrawMorseLink was written by Siddarth Sankaran
at the University of Toronto in the summer of 2005.



```
SubLink[#, {1, 2}] & /@ Ls
```

```
{PD[X[3, 1, 4, 2], X[2, 4, 1, 3]], PD[X[3, 3, 2, 2], Loop[1]]}
```

```
Length[Ls = Select[AllLinks[{2, 11}], Length[Skeleton[#]] == 3 &]]
```

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```
PD[NewLink[1]] = PD[X[1, 23, 2, 22], X[3, 24, 4, 17], X[4, 12, 5, 11],
   X[6, 13, 7, 14], X[9, 3, 10, 2], X[12, 20, 13, 19], X[14, 21, 15, 22], X[15, 8, 16, 1],
   X[17, 11, 18, 10], X[18, 5, 19, 6], X[20, 8, 21, 7], X[23, 16, 24, 9]];
AppendTo[Ls, NewLink[1]];
```

```
trivials = {-1/Sqrt[q] - Sqrt[q], 1, -1/q^2 - 1/q, -q - q^2}
```

$$\left\{-\frac{1}{\sqrt{q}}-\sqrt{q},\ 1,\ -\frac{1}{q^2}-\frac{1}{q},\ -q-q^2\right\}$$

```
Bs = Select[Ls, And[
     MemberQ[trivials, Jones[SubLink[#, {1, 2}]][q]],
     MemberQ[trivials, Jones[SubLink[#, {2, 3}]][q]],
     MemberQ[trivials, Jones[SubLink[#, {3, 1}]][q]]
    ] &]
```

```
{Link[6, Alternating, 4], Link[10, Alternating, 140],
 Link[11, Alternating, 434], Link[11, NonAlternating, 436], NewLink[1]}
```

```
trivialAs = {0};
Select[Ls, And[
    MemberQ[trivialAs, MultivariableAlexander[SubLink[#, {1, 2}]][t]],
    MemberQ[trivialAs, MultivariableAlexander[SubLink[#, {2, 3}]][t]],
    MemberQ[trivialAs, MultivariableAlexander[SubLink[#, {3, 1}]][t]]
   ] &]
```

KnotTheory::credits :

The multivariable Alexander program "MVA2" was written by Jana Archibald at the University of Toronto in 2007–2008.

First::first : {} has a length of zero and no first element. ≫

ReplacePart::psl :

Position specification {3, {}} in ReplacePart[{{1, 0}, {0, 1}, {0, 0}, {0, 0}}, 1, {3, {}}] is not an integer or a list of integers. ≫

First::first : {} has a length of zero and no first element. ≫

ReplacePart::partw : Part {4, {}} of ReplacePart[{{1, 0}, {0, 1}, {0, 0}, {0, 0}}, 1, {3, {}}] does not exist. ≫

Part::partw : Part 3 of Loop[1] does not exist. ≫

Part::partw : Part 1 of {} does not exist. ≫

Delete::psl : Position specification {}[[1, 1]] in Transpose[ReplacePart[ReplacePart[{{1, 0}, {0, 1}, {0, 0}, {0, 0}}, 1, {3, {}}], 1, {4, {}}]].{{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}.ReplacePart[ReplacePart[{{1, 0}, {0, 1}, {0, 0}, {0, 0}}, 1, {3, {}}], 1, {4, {}}] is not an integer or a list of integers. ≫

Part::partw : Part 3 of Loop[1] does not exist. ≫

General::stop : Further output of Part::partw will be suppressed during this calculation. ≫

Delete::psl : Position specification {}[[1, 1]] in Transpose[Delete[Transpose[ReplacePart[ReplacePart[{{≪2≫}, {≪2≫}, {≪2≫}, {≪2≫}}, 1, {3, {}}], 1, {4, {}}]].{{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}.ReplacePart[ReplacePart[{{1, 0}, {0, 1}, {0, 0}, {0, 0}}, 1, {3, {}}], 1, {4, {}}], {}[[1, 1]]]] is not an integer or a list of integers. ≫

Set::shape : Lists {{KnotTheory`MVA2`ni$7480, KnotTheory`MVA2`na$7480}} and {} are not the same shape. ≫

Part::pspec : Part specification KnotTheory`MVA2`ni$7480 is neither an integer nor a list of integers. ≫

Part::pspec : Part specification KnotTheory`MVA2`ni$7480 is neither an integer nor a list of integers. ≫

Set::shape : Lists {{KnotTheory`MVA2`ni$7487, KnotTheory`MVA2`na$7487}} and {} are not the same shape. ≫

Part::pspec : Part specification KnotTheory`MVA2`ni$7487 is neither an integer nor a list of integers. ≫

General::stop : Further output of Part::pspec will be suppressed during this calculation. ≫

Set::shape : Lists {{KnotTheory`MVA2`ni$7494, KnotTheory`MVA2`na$7494}} and {} are not the same shape. ≫

General::stop : Further output of Set::shape will be suppressed during this calculation. ≫

ReplacePart::psl : Position specification {}⟦1, 1⟧ in

ReplacePart[{1/4 (−1 − KnotTheory`MVA2`na$7480 + {0, Null}⟦KnotTheory`MVA2`ni$7480⟧), 0}, −1 + {1/4 (−1 − KnotTheory`MVA2`na$7480 + {≪2≫}⟦KnotTheory`MVA2`ni$7480⟧), 0}⟦{}⟦1, 1⟧⟧, {}⟦1, 1⟧]

is not an integer or a list of integers. ≫

ReplacePart::psl : Position specification {}⟦1, 1⟧ in

ReplacePart[ReplacePart[{1/4 (−1 − KnotTheory`MVA2`na$7480 + {0, Null}⟦KnotTheory`MVA2`ni$7480⟧), 0}, −1 + {1/4 (−1 + Times[≪2≫] + Part[≪2≫]), 0}⟦{}⟦1, 1⟧⟧, {}⟦1, 1⟧], −1 + ReplacePart[{1/4 (−1 + Times[≪2≫] + Part[≪2≫]), 0}, −1 + {Times[≪2≫], 0}⟦{}⟦1, 1⟧⟧, {}⟦1, 1⟧]⟦{}⟦1, 1⟧⟧, {}⟦1, 1⟧] is not an integer or a list of integers. ≫

General::stop : Further output of ReplacePart::psl will be suppressed during this calculation. ≫

First::first : {} has a length of zero and no first element. ≫

General::stop : Further output of First::first will be suppressed during this calculation. ≫

Delete::partw : Part {2} of

Transpose[Transpose[ReplacePart[{{1, 0}, {0, 1}, {0, 1}, {0, 0}}, 1, {4, {}}]].ReplacePart[{{1, 0}, {0, 1}, {0, 1}, {0, 0}}, 1, {4, {}}]] does not exist. ≫

Delete::partw : Part {2} of

Transpose[Transpose[ReplacePart[{{1, 0}, {0, 1}, {0, 1}, {0, 0}}, 1, {4, {}}]].ReplacePart[{{1, 0}, {0, 1}, {0, 1}, {0, 0}}, 1, {4, {}}]] does not exist. ≫

Delete::partw : Part {2} of

Transpose[Transpose[ReplacePart[{{1, 0}, {0, 1}, {0, 1}, {0, 0}}, 1, {4, {}}]].ReplacePart[{{1, 0}, {0, 1}, {0, 1}, {0, 0}}, 1, {4, {}}]] does not exist. ≫

General::stop : Further output of Delete::partw will be suppressed during this calculation. ≫

```
{Link[6, Alternating, 4], Link[10, Alternating, 140],
 Link[11, Alternating, 434], Link[11, NonAlternating, 436], NewLink[1]}
```

```
MultivariableAlexander[PD[Loop[1]]]
```

$$\frac{1}{\#1[1]-1}\ \&$$

```
MultivariableAlexander[PD[Loop[1], Loop[2]]]
```

$$\frac{1}{-1+\#1[\{\}[[1,1]]]}$$

```
  Det[Delete[Transpose[Delete[Transpose[ReplacePart[ReplacePart[{{1, 0}, {0, 1}, {0, 0},
              {0, 0}}, 1, {3, {}}]], 1, {4, {}}]].{{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0},
         {0, 0, 0, 0}}.ReplacePart[ReplacePart[{{1, 0}, {0, 1}, {0, 0}, {0, 0}}, 1, {3, {}}],
         1, {4, {}}], {}[[1, 1]]]], {}[[1, 1]]]] #1[
     1]^(1/2) ReplacePart[{(1/4) (-1-KnotTheory`MVA2`na$7480+{0,Null}[[KnotTheory`MVA2`ni$7480]]),0},-1+{(1/4) (-1-KnotTheory`MVA2`na$7480+{0,Nu

   #1[
     2]^(1/2) (-1+ReplacePart[{(1/4) (-1-KnotTheory`MVA2`na$7480+{0,Null}[[KnotTheory`MVA2`ni$7480]]),0},-1+{(1/4) (-1-KnotTheory`MVA2`na$7480+
   #1[3]^(1/2) {}[[1,1]] &
```

```
MultivariableAlexander[
 Append[PD[Knot[3, 1]], Loop[7]]
]
```

KnotTheory::loading : Loading precomputed data in PD4Knots`.

First::first : {} has a length of zero and no first element. ≫

ReplacePart::psl : Position specification {8, {}} in

ReplacePart[{{0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}, {0, 0, 0, 1}, {0, 1, 0, 0}, {1, 0, 0, 0}, {0, 0, 0, 0}}, 1, {8, {}}] is not an integer or a list of integers. ≫

Delete::partw : Part {3} of

Transpose[Transpose[ReplacePart[{{0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}, {0, 0, 0, 1}, {0, 1, 0, 0}, {1, 0, 0, 0}, {0, 0, 0, 0}}, 1, {8, {}}]].{{0, 0, 0, 0, 0, 0, 0, 0}, {-KnotTheory`MVA2`t$11348[2], 1, 0, -1 + KnotTheory`MVA2`t$11348[2], 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, ≪3≫, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}] does not exist. ≫

Part::partw : Part 2 of Loop[7] does not exist. ≫

Part::partw : Part 1 of {} does not exist. ≫

Part::pspec : Part specification {}[[1, 1]] is neither an integer nor a list of integers. ≫

Part::partw : Part 2 of Loop[7] does not exist. ≫

General::stop : Further output of Part::partw will be suppressed during this calculation. ≫

ReplacePart::psl :

Position specification {}[[1, 1]] in ReplacePart[{0, -5}, -1 + {0, -5}[[{}[[1, 1]]]], {}[[1, 1]]] is not an integer or a list of integers. ≫

Delete::partw : Part {3} of

Transpose[Transpose[ReplacePart[{{0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}, {0, 0, 0, 1}, {0, 1, 0, 0}, {1, 0, 0, 0}, {0, 0, 0, 0}}, 1, {8, {}}]].{{0, 0, 0, 0, 0, 0, 0, 0}, {-#1[2], 1, 0, -1 + #1[2], 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, ≪4≫, 0, 0}, {0, ≪7≫}, {0, -1 + #1[2], 0, 0, -#1[2], 1, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}] does not exist. ≫

Delete::partw : Part {3} of

Transpose[Transpose[ReplacePart[{{0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}, {0, 0, 0, 1}, {0, 1, 0, 0}, {1, 0, 0, 0}, {0, 0, 0, 0}}, 1, {8, {}}]].{{0, 0, 0, 0, 0, 0, 0, 0}, {-#1[2], 1, 0, -1 + #1[2], 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, ≪4≫, 0, 0}, {0, ≪7≫}, {0, -1 + #1[2], 0, 0, -#1[2], 1, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}] does not exist. ≫

General::stop : Further output of Delete::partw will be suppressed during this calculation. ≫

$$\left\{\frac{1}{-1+\#1[2]}\right.$$

```
Det[Delete[Transpose[Transpose[ReplacePart[{{0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 1, 0},
          {0, 0, 0, 1}, {0, 0, 0, 1}, {0, 1, 0, 0}, {1, 0, 0, 0}, {0, 0, 0, 0}}, 1, {8, {}}]].
      {{0, 0, 0, 0, 0, 0, 0, 0}, {-#1[2], 1, 0, -1 + #1[2], 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
       {0, 0, -#1[2], 1, 0, -1 + #1[2], 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, -1 + #1[2],
        0, 0, -#1[2], 1, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}], 3]]
```

$$\#1[2]^{2+\frac{1}{2}(-1+\{0,-5\}[\![\{\}[\![1,1]\!]\!])}\ \#1[3]^{\frac{1}{2}\{\}[\![1,1]\!]},\ \frac{1}{\#1[1]^{5/2}\ (-1+\#1[2])}$$

```
Det[Delete[Transpose[Transpose[ReplacePart[{{0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 1, 0},
          {0, 0, 0, 1}, {0, 0, 0, 1}, {0, 1, 0, 0}, {1, 0, 0, 0}, {0, 0, 0, 0}}, 1, {8, {}}]].
      {{0, 0, 0, 0, 0, 0, 0, 0}, {-#1[2], 1, 0, -1 + #1[2], 0, 0, 0, 0},
       {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, -#1[2], 1, 0, -1 + #1[2], 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
       {0, -1 + #1[2], 0, 0, -#1[2], 1, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
       {0, 0, 0, 0, 0, 0, 0, 0}}], 3]] #1[2]^(2+1/2 (-1+{0,-5}[[{}[[1,1]]])) #1[3]^(1/2 {}[[1,1]])} &
```