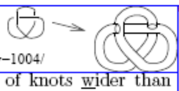
Luminy Z2A Talk

December-17-09
12:08 PM

w−Knots from Z to A

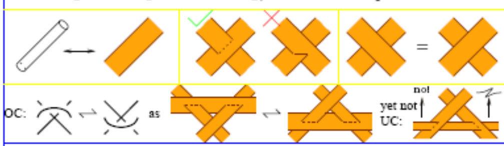
Dror Bar−Natan, Luminy, April 2010
http://www.math.toronto.edu/~drorbn/Talks/Luminy−1004/



Abstract I will define w-knots, a class of knots wider than ordinary knots but weaker than virtual knots, and show that it is quite easy to construct a universal finite invariant Z of w-knots. In order to study Z we will introduce the "Euler Operator" and the "Infinitesimal Alexander Module", at the end finding a simple determinant formula for Z. With no doubt that formula computes the Alexander polynomial A, except I don't have a proof yet.

Tubes in 4D.

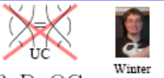
A Ribbon 2-Knot is a surface S embedded in $\mathbb{R}^4$ that bounds an immersed handlebody B, with only "ribbon singularities"; a ribbon singularity is a disk D of trasverse double points, whose preimages in B are a disk D_1 in the interior of B and a disk D_2 with $D_2 \cap \partial B = \partial D_2$, modulo isotopies of S alone.



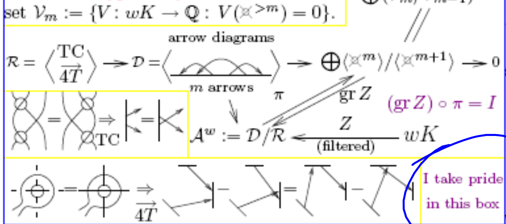
w−Knots.

$wK = CA\langle \ \rangle / $ R23, OC

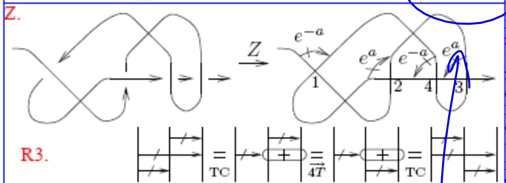
$= PA\langle \ \rangle / $ R23, VR123, D, OC



The Finite Type Story. With $\times := \times - \times$ set $\mathcal{V}_m := \{V : wK \to \mathbb{Q} : V(\times^{>m}) = 0\}$.

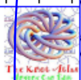


Z.

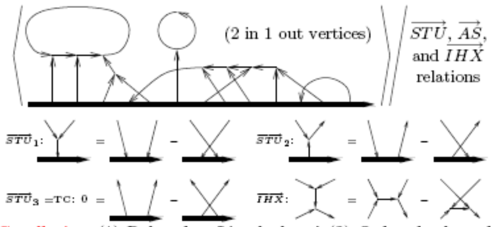


"God created the knots, all else in topology is the work of mortals."
Leopold Kronecker (modified)

www.katlas.org



The Bracket-Rise Theorem. $\mathcal{A}^w$ is isomorphic to

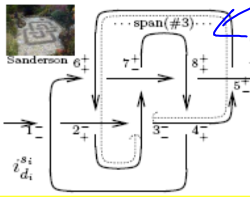


Corollaries. (1) Related to Lie algebras! (2) Only wheels and isolated arrows persist.

Habiro - can you do better?

all that's 3 should be blue

The Alexander Theorem.



$T_{ij} = |\mathrm{low}(\#j) \in \mathrm{span}(\#i)|$,
$s_i = \mathrm{sign}(\#i)$, $d_i = \mathrm{dir}(\#i)$,
$S = \mathrm{diag}(s_i d_i)$,
$A = \det\left(I + T(I - X^{-S})\right)$.

$$T = \begin{pmatrix} 0&1&1&1&1&0&1&0 \\ 0&0&1&0&1&0&0&0 \\ 0&1&0&0&1&0&0&0 \\ 0&1&0&0&1&0&1&0 \\ 0&1&0&1&0&1&1&1 \\ 0&1&0&1&0&0&1&0 \\ 0&0&0&1&0&1&0&0 \\ 0&0&0&1&0&1&0&0 \end{pmatrix},$$

$X^{-S} = \mathrm{diag}(\frac{1}{X}, X, \frac{1}{X}, X, X, \frac{1}{X}, X, \frac{1}{X})$.

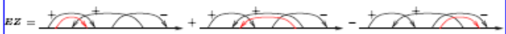
space

Conjecture. For u-knots, A is the Alexander polynomial.
Theorem. With $w : x^k \mapsto w_k =$ (the k-wheel),

$$Z = N \exp_{\mathcal{A}^w}\left(-w\left(\log_{\mathbb{Q}[\![x]\!]} A(e^x)\right)\right)$$

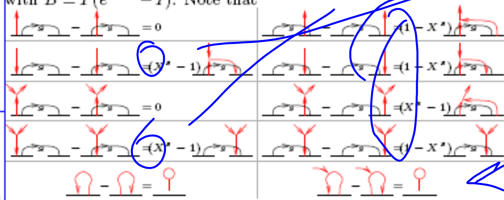
mod $w_k w_l = w_{k+l}$, $Z = N \cdot A^{-1}(e^x)$

Proof Sketch. Let E be the Euler operator, "multiply anything by its degree", $f \mapsto xf'$ in $\mathbb{Q}[\![x]\!]$, so $Ee^x = xe^x$ and



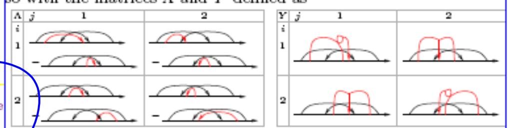
We need to show that $Z^{-1}EZ = N' - \mathrm{tr}\left((I-B)^{-1}TSe^{-xS}\right)w_1$, with $B = T(e^{-xS} - I)$. Note that

align



something about hair

so with the matrices Λ and Y defined as



we have $EZ - N'' = \mathrm{tr}(S\Lambda)$, $\Lambda = -BY - Te^{-xS}w_1$, and $Y = BY + Te^{-xS}w_1$. The theorem follows.

So What? • Habiro-Shima did this already, but not quite. (HS: *Finite Type Invariants of Ribbon 2-Knots, II,* Top. and its Appl. **111-3** (2001) 265–287.)

Z

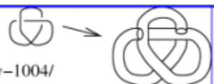
• New (?) formula for Alexander, new (?) "Infinitesimal Alexander Module".
• An "ultimate Alexander invariant": local, composes well, behaves under cabling. Ought to also generalize the multi-variable Alexander polynomial and the theory of Milnor linking numbers.
• Tip of the Alekseev-Torossian-Kashiwara-Vergne iceberg (AT: *The Kashiwara-Vergne conjecture and Drinfeld's associators,* arXiv:0802.4300).
• Tip of the v-knots iceberg.
Also see http://www.math.toronto.edu/˜drorbn/papers/WKO/

connects Lescop

green

w−Knots from Z to A

Dror Bar−Natan, Luminy, April 2010
http://www.math.toronto.edu/~drorbn/Talks/Luminy−1004/



Abstract I will define w-knots, a class of knots wider than ordinary knots but weaker than virtual knots, and show that it is quite easy to construct a universal finite invariant Z of w-knots. In order to study Z we will introduce the "Euler Operator" and the "Infinitesimal Alexander Module", at the end finding a simple determinant formula for Z. With no doubt that formula computes the Alexander polynomial A, except I don't have a proof yet.

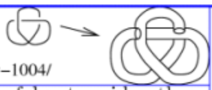
Tubes in 4D. Broken surface

The Alexander Thm

w–Knots from Z to A

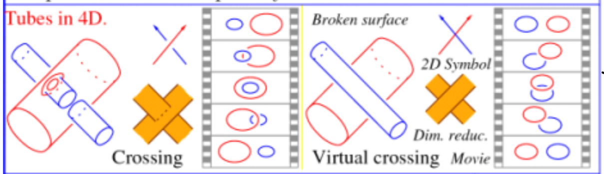
Dror Bar–Natan, Luminy, April 2010

http://www.math.toronto.edu/~drorbn/Talks/Luminy-1004/

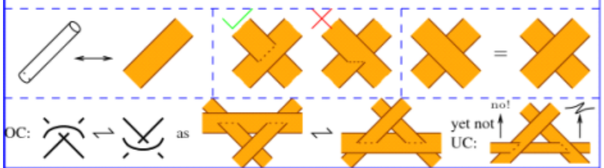


Abstract I will define w-knots, a class of knots wider than ordinary knots but weaker than virtual knots, and show that it is quite easy to construct a universal finite invariant Z of w-knots. In order to study Z we will introduce the "Euler Operator" and the "Infinitesimal Alexander Module", at the end finding a simple determinant formula for Z. With no doubt that formula computes the Alexander polynomial A, except I don't have a proof yet.

Tubes in 4D.



A Ribbon 2-Knot is a surface S embedded in $\mathbb{R}^4$ that bounds an immersed handlebody B, with only "ribbon singularities"; a ribbon singularity is a disk D of trasverse double points, whose preimages in B are a disk D_1 in the interior of B and a disk D_2 with $D_2 \cap \partial B = \partial D_2$, modulo isotopies of S alone.

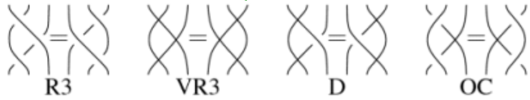


w–Knots.

→ $wK = CA\langle \cdots \rangle / \cancel{R1}23$, OC.

→ $= PA\langle \cdots \rangle / \cancel{R1}23$, VR123, D, OC.


Winter



The F.T. story

Bracket rise

Itabira: can you do better?

Also see http://www.math.toronto.edu/~drorbn/papers/WKO/ 28/3/10, 5:51pm

The Alexander Thm

P
r
o
o
f

* Differentiation
* bulk management
* Computation

So what?
* The ultimate Alexander invariant: local, composes well, behaves under cabling.
* Tip of the AlekTorKashVergne iceberg
* Tip of the v-knots iceberg.

$\not{X} = X' - X$, def of $\mathcal{V}_m$.

$\bigoplus(\mathcal{V}_m/\mathcal{V}_{m-1})^*$

$\|$

$\left\{\begin{matrix}OC\\ \overrightarrow{4T}\end{matrix}\right\} \longrightarrow \{\text{arrow diags! } m \text{ arrows}\} \longrightarrow \bigoplus\{m\text{-cubes: } m \text{ semi-virts}\} / \{(m+1)\text{-cubes}\} \to 0$

π, $\mathrm{gr}\, Z$

$\mathrm{gr}\, Z \circ \pi = Id$

$A = \mathcal{D}/_{OC, \overrightarrow{4T}} \xleftarrow[Z]{\text{filtered,}} \{w\text{-knots}\}$

The F.T story