Luminy 18C Talk

April-10-10
9:38 PM

R2b R2C R3b

list

v_n/v_{n-1}

flat: X = X

18 Conjectures

Dror Bar-Natan, Luminy, April 2010
http://www.math.toronto.edu/~drorbn/Talks/Luminy-1004/

Abstract I will state 18 = 3 × 3 × 2 "fundamental" conjectures on finite type invariants of various classes of virtual knots. This done, I will state a few further conjectures about these conjectures and ask a few questions about how these 18 conjectures may or may not interact.

Two tables. The following tables show dim $V_{n/n-1}$ and dim W_n for $n = 1, \ldots, 5$ for 18 classes of v-knots:

relations\skeleton		round	long	~~descending~~ flat
standard ~~R23~~	mod R1	0, 0, 1, 4, 17	0, 2, 7, 42, 246	0, 0, 1, 6, 34
	no R1	1, 1, 2, 7, 29	2, 5, 15, 67, 365	1, 1, 2, 8, 42
braid-like ~~R23~~	mod R1	0, 0, 1, 4, 17	0, 2, 7, 42, 246	0, 0, 1, 6, 34
	no R1	1, 2, 5, 19, 77	2, 7, 27, 139, 813	1, 2, 6, 24, 120
R2 only	mod R1	0, 0, 4, 44, 648	0, 2, 28, 420, 7808	0, 0, 2, 18, 174
	no R1	1, 3, 16, 160, 2248	2, 10, 96, 1332, 23880	1, 2, 9, 63, 570

18 Conjectures. ~~This coincidence persists.~~

These 18 coincidences persist.

Definitions

Comments

add a "comparison with u table".

Technique

The u–v–w Story

	u-Knots →	v-Knots →	w-Knots
Topology	Ordinary (usual) knotted objects in 3D — braids, knots, links, tangles, knotted graphs, etc.	Virtual knotted objects — "algebraic" knotted objects, or "not specifically embedded" knotted objects; knots drawn on a surface, modulo stabilization.	Ribbon knotted objects in 4D; "flying rings". Like v, but also with "overcrossings commute".
Combinatorics	Chord diagrams and Jacobi diagrams, modulo $4T$, STU, IHX, etc.	Arrow diagrams and v-Jacobi diagrams, modulo $6T$ and various "directed" STUs and IHXs, etc.	Like v, but also with "tails commute". Only "two in one out" internal vertices.
Low Algebra	Finite dimensional metrized Lie algebras, representations, and associated spaces.	Finite dimensional Lie bi-algebras, representations, and associated spaces.	Finite dimensional co-commutative Lie bi-algebras (i.e., $\mathfrak{g} \ltimes \mathfrak{g}^*$), representations, and associated spaces.
High Algebra	The Drinfel'd theory of associators.	Likely, quantum groups and the Etingof-Kazhdan theory of quantization of Lie bi-algebras.	The Kashiwara-Vergne-Alekseev-Torossian theory of convolutions on Lie groups and Lie algebras.

recycle

Dream

Some Propaganda

"God created the knots, all else in topology is the work of mortals."

Leopold Kronecker (modified)

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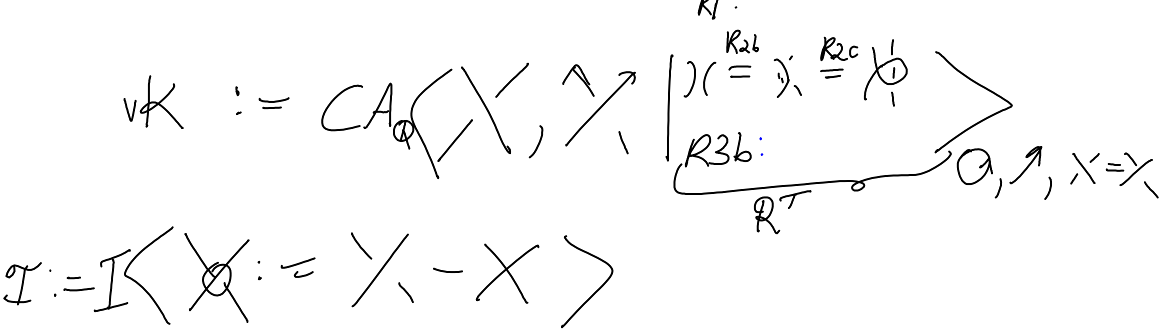
Definitions.

FlipFlop	mem card



$V_n = (vK/I^{n+1})^*$ is one thing we measure.

$\mathcal{V}_n = (vK/\mathcal{I}^{n+1})^*$ is one thing we measure....

$\mathcal{V}_n/\mathcal{V}_{n-1} = (\mathcal{I}^n/\mathcal{I}^{n+1})^*$

"arrow diagrams"

$$\mathcal{R}^H_n \longrightarrow \mathcal{D}_n = \underbrace{CA\langle |\overset{+}{\to}|, |\overset{-}{\to}| \rangle_n}_{\text{exact ?}} \qquad \begin{matrix} \overset{+}{\mapsto} \xrightarrow{\tau} \times = \dots \\ \overset{-}{\mapsto} \xrightarrow{\tau} \times = \dots \end{matrix} \longrightarrow \mathcal{I}^n/\mathcal{I}^{n+1}$$

$$\mathcal{R}^H = \left\{ \begin{matrix} R1 \longrightarrow \\ R2b \longrightarrow \\ R2c \longrightarrow \\ R3b \longrightarrow \\ \times' = \times \longrightarrow \end{matrix} \right\}$$

$\mathcal{W}_n = (\mathcal{D}_n/\mathcal{R}_n)^*$ is the other thing we measure

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The Polyak Technique:

fails in the u case.

$$vK = CA_{\otimes}\langle \times, \times \rangle / \mathcal{R}^o = \left\{ \begin{matrix} 8T: \\ etc. \end{matrix} \right\}$$

flip looks right

$$vK/\mathcal{I}^{n+1} = CA_{\otimes}^{\leq n}\langle \times', \times' \rangle / \mathcal{R}^{o \leq n}$$

This is a finite, computable space!

$$\mathcal{I}^n/\mathcal{I}^{n+1} \longleftarrow \mathcal{D}_n \longleftarrow \left\{ \begin{matrix} \text{degree } n \\ \text{"bottoms"} \\ \text{of rels in } \mathcal{R}^o \end{matrix} \right\} = \tau \mathcal{R}^H_n$$