Goettingen - Sunday preps

April-25-10
7:15 AM

Day 1 – u, v, w: topology and philosophy

u, v, and w–Knots: Topology, Combinatorics and Low and High Algebra

Dror Bar–Natan, Goettingen, April 2010

http://www.math.toronto.edu/~drorbn/Talks/Goettingen-1004/

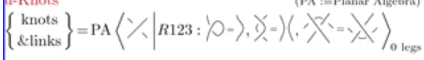
Plans and Dreams

$$\begin{pmatrix}\text{arbitrary algebraic}\\ \text{structure}\end{pmatrix} \xrightarrow[\text{machine}]{\text{projectivization}} \begin{pmatrix}\text{a problem in}\\ \text{graded algebra}\end{pmatrix}$$

- Feed knot-things, get Lie algebra things.
- Feed u-knots, get Drinfel'd associators.
- Feed w-knots, get Kashiware-Vergne-Alekseev-Torossian.
- Dream: Feed v-knots, get Etingof-Kazhdan.
- Dream: Knowing the question whose answer is 42, or E-K, will be useful to algebra and topology.

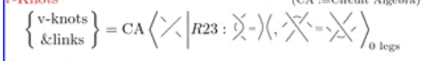
u-Knots

(PA :=Planar Algebra)

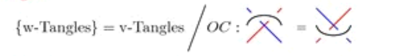


v-Knots

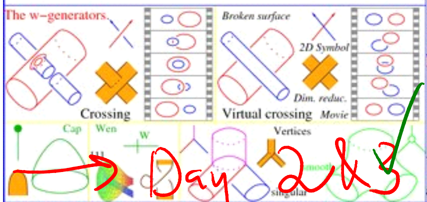
(CA :=Circuit Algebra)



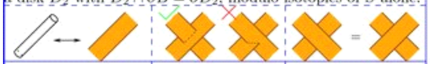
w-Tangles



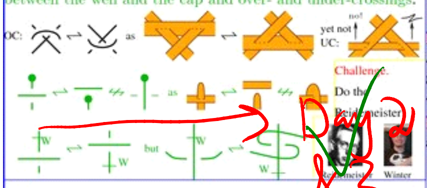
The w–generators.



A Ribbon 2-Knot is a surface S embedded in $\mathbb{R}^4$ that bounds an immersed handlebody B, with only "ribbon singularities"; a ribbon singularity is a disk D of trasverse double points, whose preimages in B are a disk D_1 in the interior of B and a disk D_2 with $D_2 \cap \partial B = \partial D_2$, modulo isotopies of S alone.

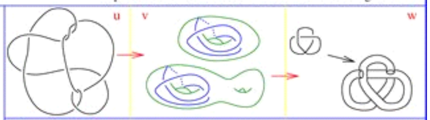


The w-relations include R234, VR1234, M, Overcrossings Commute (OC) but not UC, $W^2 = 1$, and funny interactions between the wen and the cap and over- and under-crossings:

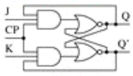


[1] http://qlink.queensu.ca/~4lb11/interesting.html

Also see http://www.math.toronto.edu/~drorbn/papers/WKO/



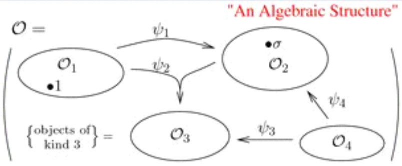
Circuit Algebras



A J-K Flip Flop

Infineon HYS64T64020HDL-3.7-A 512MB RAM

"An Algebraic Structure"



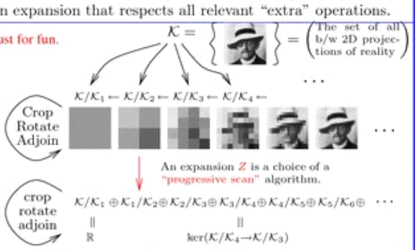
- Has kinds, objects, operations, and maybe constants.
- Perhaps subject to some axioms.
- We always allow formal linear combinations.

Homomorphic expansions for a filtered algebraic structure $\mathcal{K}$:

$$\begin{array}{l} \text{ops} \circlearrowright \mathcal{K} = \mathcal{K}_0 \supset \mathcal{K}_1 \supset \mathcal{K}_2 \supset \mathcal{K}_3 \supset \dots \\ \quad\Downarrow \qquad\qquad\qquad\quad \downarrow Z \\ \text{ops} \circlearrowright \operatorname{gr}\mathcal{K} := \mathcal{K}_0/\mathcal{K}_1 \oplus \mathcal{K}_1/\mathcal{K}_2 \oplus \mathcal{K}_2/\mathcal{K}_3 \oplus \mathcal{K}_3/\mathcal{K}_4 \oplus \dots \end{array}$$

An expansion is a filtration respecting $Z : \mathcal{K} \to \operatorname{gr}\mathcal{K}$ that "covers" the identity on $\operatorname{gr}\mathcal{K}$. A homomorphic expansion is an expansion that respects all relevant "extra" operations.

Just for fun.



Filtered algebraic structures are cheap and plenty. In any $\mathcal{K}$, allow formal linear combinations, let $\mathcal{K}_1$ be the ideal generated by differences (the "augmentation ideal"), and let $\mathcal{K}_m := \langle (\mathcal{K}_1)^m \rangle$ (using all available "products").

Examples. 1. The projectivization of a group is a graded associative algebra. 2. Quandle: a set Q with an op $\wedge$ s.t.

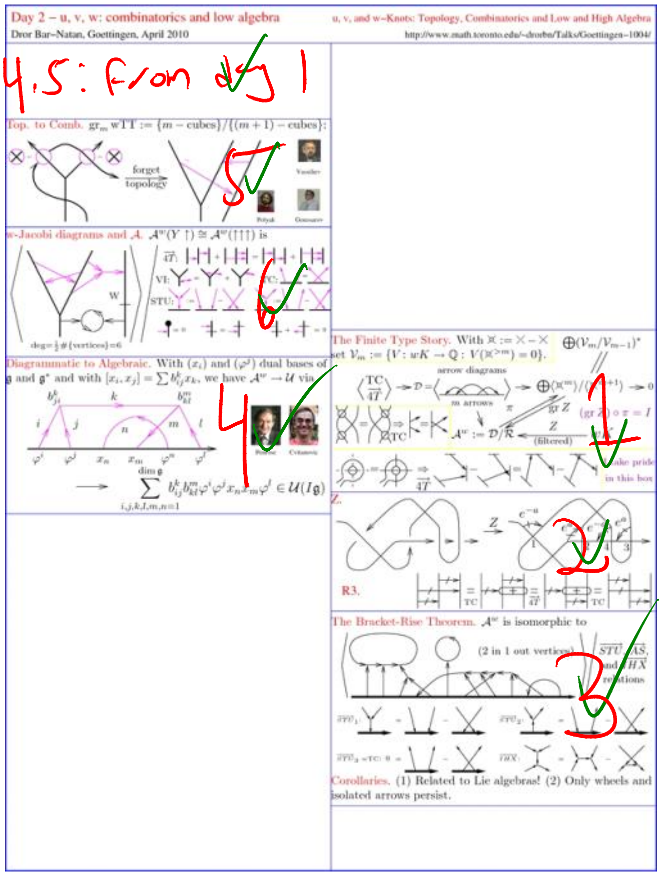
$$1 \wedge x = 1, \quad x \wedge 1 = x, \qquad \text{(appetizers)}$$
$$(x \wedge y) \wedge z = (x \wedge z) \wedge (y \wedge z). \qquad \text{(main)}$$

$\operatorname{proj} Q$ is a graded Leibniz algebra: Roughly, set $\bar{v} := (v-1)$ (these generate I!), feed $1+\bar{x}$, $1+\bar{y}$, $1+\bar{z}$ in (main), collect the surviving terms of lowest degree:

$$(\bar{x} \wedge \bar{y}) \wedge \bar{z} = (\bar{x} \wedge \bar{z}) \wedge \bar{y} + \bar{x} \wedge (\bar{y} \wedge \bar{z}).$$

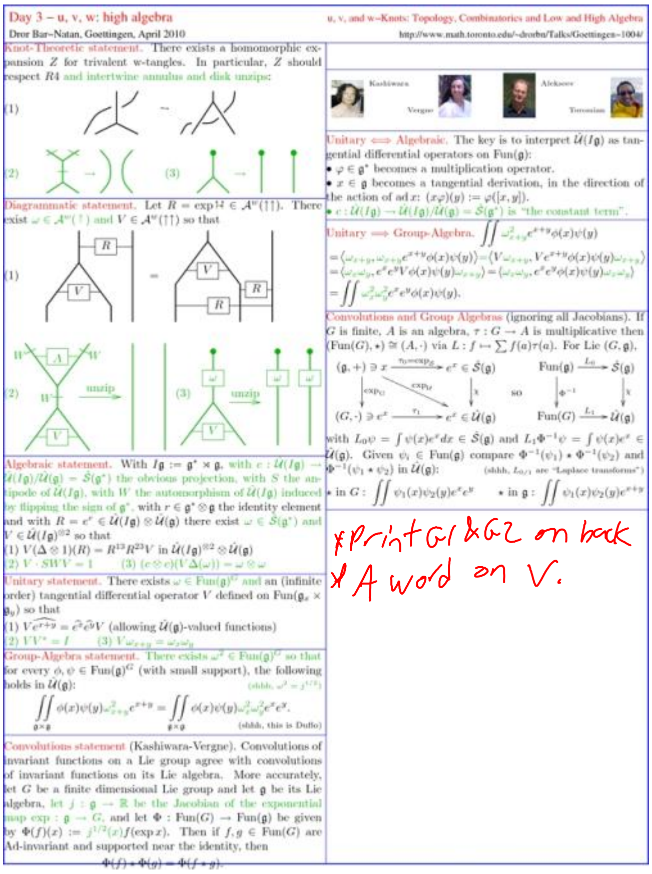
add: preview of day 2.

* Add a $\mathcal{K} \xrightarrow[\text{high}]{Z} * \xrightarrow[\text{low}]{} \mathcal{L}ie$ box.



- Finite type invariants, weight systems, chord diagrams, arrow diagrams, 4T relations
- The "bracket-rise" theorem, STU and IHX relations
- Maps into various kinds of universal enveloping algebras.

*all arrows go purple.



- Kashiwara-Vergne and Alekseev-Torossian: convolutions, integrals, measure preserving transformations, unitary operators, universal formulas and universal equations.
- A word on knotted trivalent graphs, Drinfel'd associators, and Chern-Simons-Witten theory.
- Dreams on v-knots, Etingof-Kazhdan, and quantization of Lie bi-algebras.
- Hallucinations on knot homologies and on further physics.

* ~~Bring material from day 1~~

*Print G1 & G2 on back

*A word on V.

ouch.