

**Day 1 - u, v, w: topology and philosophy**  
Dror Bar-Natan, Goettingen, April 2010  
http://www.math.toronto.edu/~bar/Goettingen-1004

**u, v, and w-Knots: Topology, Combinatorics and Low and High Algebra**  
http://www.math.toronto.edu/~bar/Goettingen-1004

**Circuit Algebras**  
A J-K Flip Flop

**What are v-Trivalent Tangles?**  
(PA = Planar Algebras)  
knots & links = PA( $\langle \text{R123}, \dots \rangle$ )  
v-trivalent tangle = PA( $\langle \text{R23}, \text{R4} \rangle$ )  
w-tangles = PA( $\langle \text{generators}, \text{relations} \rangle$ )

**The w-generators**  
Broken surface  
Crossing  
Virtual crossing  
w-relations include R234, VR1234, M, Overcrossings, Commute (OC) but not UC,  $\mathbb{H}^2 = 1$ , and funny interactions between the wcc and the cap and over- and under-crossings.

**Filtered algebraic structures are cheap and plenty.**  
Examples: 1. The projectivization of a group is a graded associative algebra. 2. Quasideterminants are a set  $\{Q_i\}$  of an op- $\mathcal{A}$  s.t.  $1 \otimes x = 1, x \otimes 1 = 1$ .  
 $(x \otimes y) \otimes z = (x \otimes (y \otimes z))$  (associativity)  
 $(x \otimes y) \otimes z = (x \otimes y) \otimes z$  (triviality)  
 $(x \otimes y) \otimes z = (x \otimes y) \otimes z$  (triviality)  
 $(x \otimes y) \otimes z = (x \otimes y) \otimes z$  (triviality)

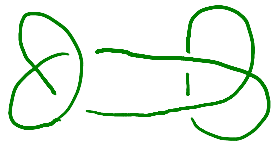
**Handwritten notes:**  
→ Dreams and plans  
→ vob →  
→ Rewrite  
→ add a v-knot example and a v-knot example.  
→ will be useful to algebra and topology.

- Dreams and plans.
- Knots, planar diagrams, Reidemeister moves, virtual knots are to knots as manifolds are to Euclidean spaces, flying rings and knotted tubes in 4D and w-knots.
- Planar algebras and circuit algebras.
- The abstract machine - filtered and graded spaces, expansions and homomorphic expansions, equations in graded spaces.

**Dreams and plans:**



1. Feed knot-things, get Liably things
2. Feed w-knots, get Drinfeld associators.
3. Feed w-knots, get KVAT
4. Dream: Feed v-knots, get Ek
5. Dream: knowing the question whose answer is 42 (or Ek)



knots on a surface.