Derivations and Modules

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FA_n is an $\mathcal{A}^{w/w}$ module.... there ought to be a direct way to get out of this that $P\mathcal{A}^{w/w}$ acts on FL_n by derivations.

In general, if a Lie algebra $\mathfrak{g}$ acts on a Lie algebra L by derivations, does this make $\mathcal{U}(L)$ a $\mathcal{U}(\mathfrak{g})$-module?

$$g_1 g_2 g_3 \cdots g_n \cdot l_1 l_2 \dots l_k \longrightarrow \text{the obvious sum.}$$

(need to know that $\mathfrak{g} \xrightarrow{b} \mathrm{Der}(L)$ is a Lie algebra morphism)

Can we invert this? That is, when is it that a $\mathcal{U}(\mathfrak{g})$-module comes from this construction?

$$\begin{array}{ccc} \mathcal{U}(\mathfrak{g}) \otimes L \otimes L & \xrightarrow{\sigma_{23} \circ \Delta} & \mathcal{U}(\mathfrak{g}) \otimes L \otimes \mathcal{U}(\mathfrak{g}) \otimes L \\ \downarrow 1 \otimes [,] & \circlearrowright & \downarrow b \otimes b \\ \mathcal{U}(\mathfrak{g}) \otimes L & & L \otimes L \\ & \searrow^{b} & \downarrow [,] \\ & & L \end{array}$$

This is the $\mathcal{U}(\mathfrak{g})$-version of the statement "$\mathfrak{g}$ acts by derivations"!

$\Longrightarrow$ Can be generalized to an arbitrary co-algebra acting on a Lie algebra.