What's R-Torsion?

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4:01 PM

$(\mathcal{C}, b) \xrightarrow{\tau} \tau(\mathcal{C},b) \in \Lambda^{top}(H^{even}) \otimes \Lambda^{top}(H^{odd *})$

(complex → $\mathcal{C}$; bases → b)

A category like $\{\mathcal{C}\}$ is a category, with trivial addition of bases.

$$C^1 \to C^2 \to C^3 \to C^4$$

$d: A \to B$ gives an iso $A/\ker d \to \operatorname{im} d$

and so a canonical element of $\Lambda^{top}(A/\ker d) \otimes \Lambda^{top}(\operatorname{im} d)$

and so a map

$$\Lambda^{top}(A) \otimes \Lambda^{-top}(B) \to \Lambda^{top} H_A \otimes \Lambda^{-top} H_B$$

So in general $\tau: \Lambda^{top}(\mathcal{C}) \to \Lambda^{top}(H)$ } i.e., given a fixed dimension profile, $\dim C^r = n_r$

w/ fixed dim profile.

$$\begin{array}{ccc} \mathcal{C}_1 & \Lambda^{top}(\mathcal{C}_1) & \xrightarrow{\tau} \Lambda^{top}(H_1) \\ \downarrow F & \downarrow F & \downarrow F \\ \mathcal{C}_2 & \Lambda^{top}(\mathcal{C}_2) & \xrightarrow{\tau} \Lambda^{top}(H_2) \end{array}$$

So τ is a natural transformation.

Is it homotopy invariant?

$$\begin{array}{ccc} \mathcal{C}_1 & \Lambda^{top}(\mathcal{C}_1) & \xrightarrow{\tau} \Lambda^{top}(H_1) \\ F \Big(\overset{h}{\Rightarrow}\Big) G & F \Big(\quad \Big) G & F \Big(=\Big) G \\ \mathcal{C}_2 & \Lambda^{top}(\mathcal{C}_2) & \xrightarrow{\tau} \Lambda^{top}(H_2) \end{array}$$

$G_2 \qquad \Lambda^{top}(\dot{G}_2) \xrightarrow{\tau} \Lambda^{top}(\dot{H}_2)$