

Isgur@GSS: Self-Referencing Recursions and Labeled Binary Trees

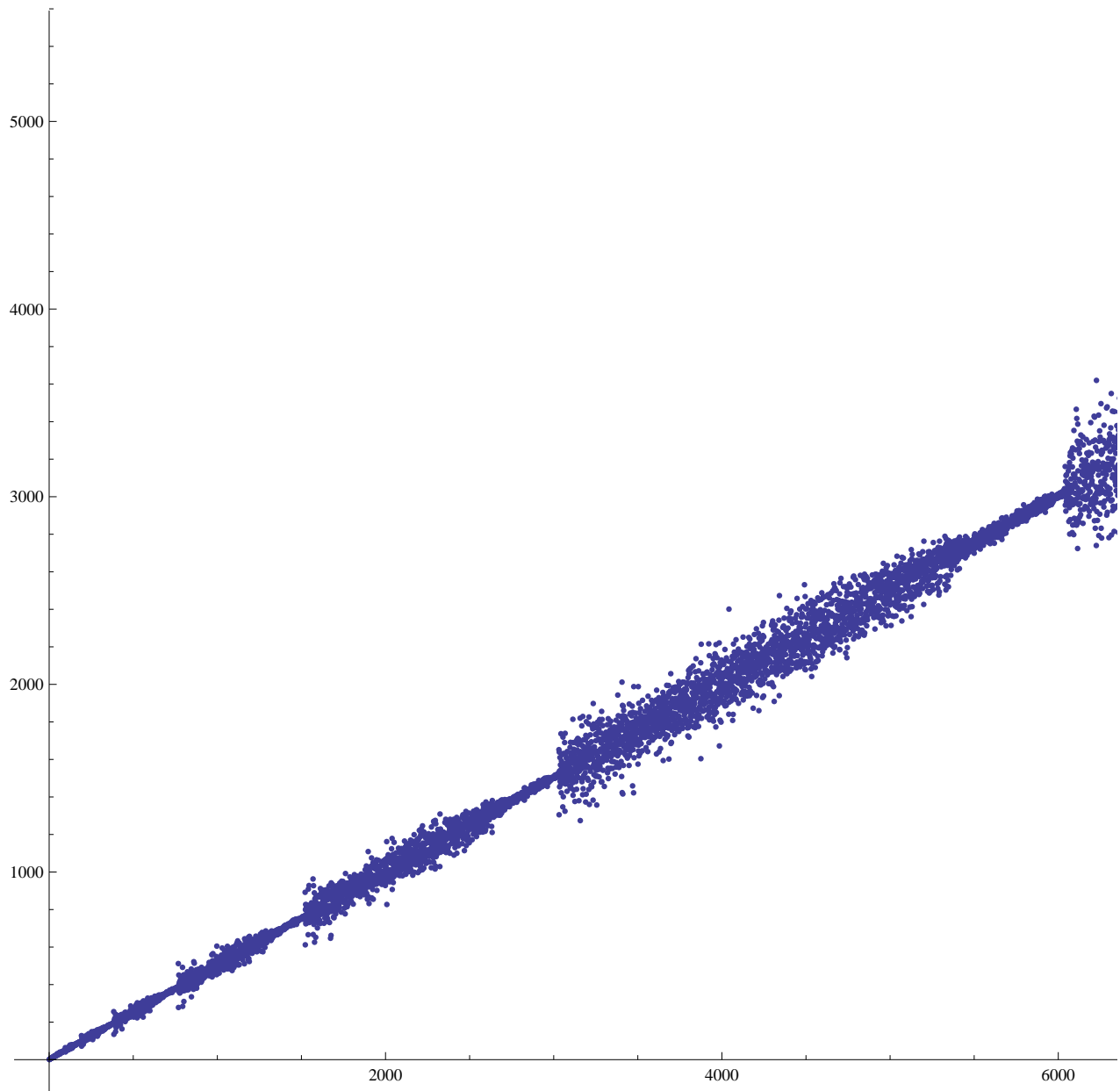
Hofstadter's Recursion

```
Clear[Q];
Q[1] = Q[2] = 1;
Q[n_] /; n > 2 := Q[n] = Q[n - Q[n - 1]] + Q[n - Q[n - 2]]

Q /@ Range[200]

{1, 1, 2, 3, 3, 4, 5, 5, 6, 6, 6, 8, 8, 8, 10, 9, 10, 11, 11, 12, 12, 12, 12, 16, 14, 14,
16, 16, 16, 16, 20, 17, 17, 20, 21, 19, 20, 22, 21, 22, 23, 23, 24, 24, 24, 24, 24, 32,
24, 25, 30, 28, 26, 30, 30, 28, 32, 30, 32, 32, 32, 32, 40, 33, 31, 38, 35, 33, 39, 40,
37, 38, 40, 39, 40, 39, 42, 40, 41, 43, 44, 43, 43, 46, 44, 45, 47, 47, 46, 48, 48, 48,
48, 48, 48, 64, 41, 52, 54, 56, 48, 54, 54, 50, 60, 52, 54, 58, 60, 53, 60, 60, 52, 62,
66, 55, 62, 68, 62, 58, 72, 58, 61, 78, 57, 71, 68, 64, 63, 73, 63, 71, 72, 72, 80, 61,
71, 77, 65, 80, 71, 69, 77, 75, 73, 77, 79, 76, 80, 79, 75, 82, 77, 80, 80, 78, 83, 83,
78, 85, 82, 85, 84, 84, 88, 83, 87, 88, 87, 86, 90, 88, 87, 92, 90, 91, 92, 92, 94, 92,
93, 94, 94, 96, 94, 96, 96, 96, 96, 96, 96, 128, 72, 96, 115, 100, 84, 114, 110, 93}
```

```
ListPlot[Q /@ Range[10 000]]
```



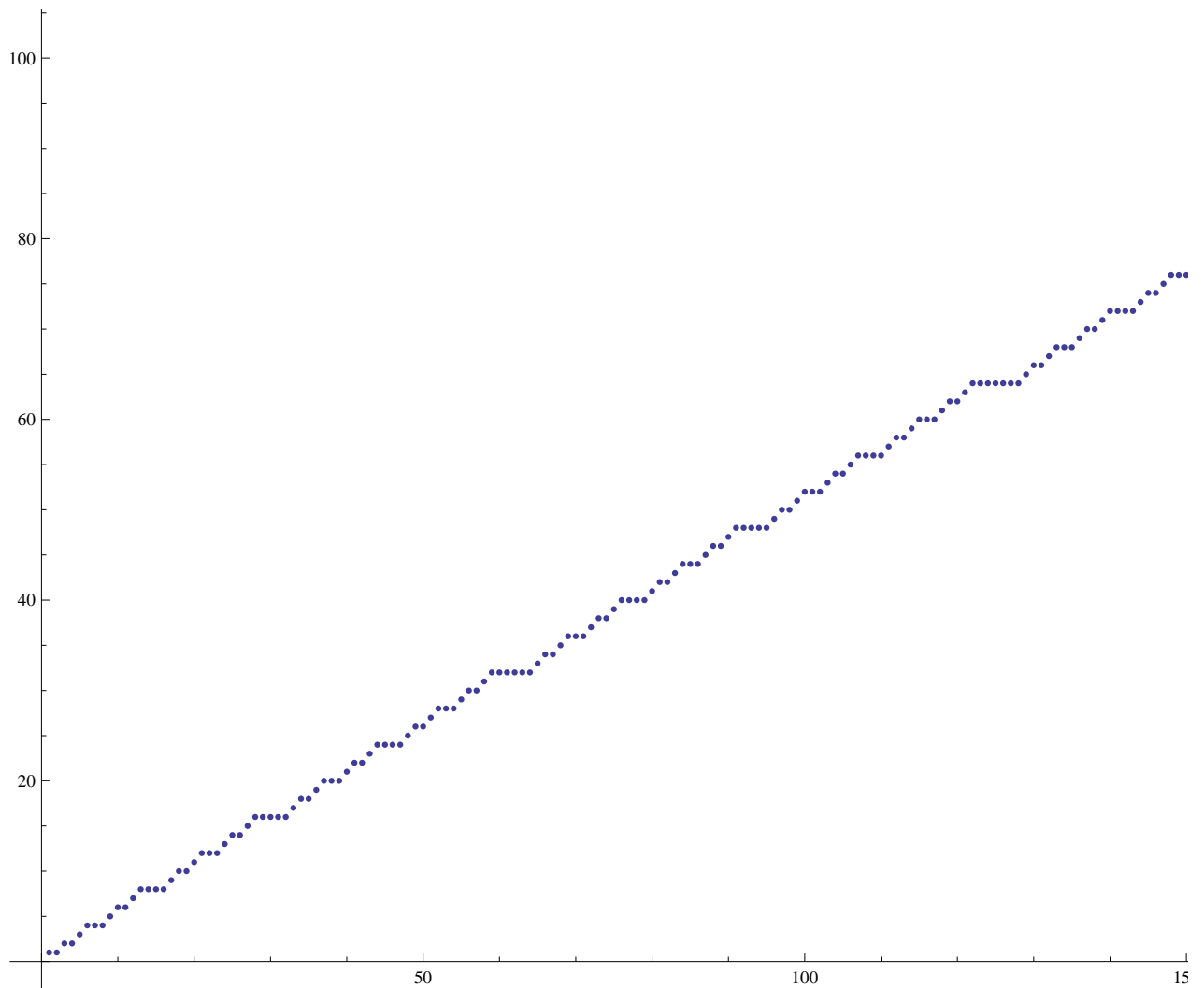
Conolly's Recursion

```
Clear[CC];
CC[1] = CC[2] = 1;
CC[n_] /; n > 2 := CC[n] = CC[n - CC[n - 1]] + CC[n - 1 - CC[n - 2]]

CC /@ Range[100]

{1, 1, 2, 2, 3, 4, 4, 4, 5, 6, 6, 7, 8, 8, 8, 8, 9, 10, 10, 11, 12, 12, 12, 13, 14, 14, 15, 16, 16,
16, 16, 16, 17, 18, 18, 19, 20, 20, 20, 21, 22, 22, 23, 24, 24, 24, 24, 25, 26, 26, 27, 28, 28,
28, 29, 30, 30, 31, 32, 32, 32, 32, 32, 32, 33, 34, 34, 35, 36, 36, 36, 37, 38, 38, 39, 40, 40,
40, 40, 41, 42, 42, 43, 44, 44, 44, 45, 46, 46, 47, 48, 48, 48, 48, 48, 48, 49, 50, 50, 51, 52}
```

```
ListPlot[CC /@ Range[200]]
```



Golomb's Recursion

```
Clear[G];
G[1] = G[2] = 1;
G[n_] /; n > 2 := G[n] = G[n - G[n - 1]] + G[n - 1 - G[n - 2]] + 1
G /@ Range[100]
{1, 1, 3, 3, 3, 5, 5, 5, 7, 7, 7, 7, 9, 9, 9, 11, 11, 11, 11, 13, 13, 13, 15, 15, 15, 15, 15, 17, 17,
17, 19, 19, 19, 19, 21, 21, 21, 23, 23, 23, 23, 23, 25, 25, 25, 27, 27, 27, 27, 29, 29, 29, 31,
31, 31, 31, 31, 33, 33, 33, 35, 35, 35, 35, 37, 37, 37, 39, 39, 39, 39, 39, 41, 41, 41, 43,
43, 43, 43, 45, 45, 45, 47, 47, 47, 47, 47, 47, 49, 49, 49, 51, 51, 51, 51, 53, 53, 53, 55}
```

ListPlot[G /@ Range[200]]

