The Kauffman Polynomial

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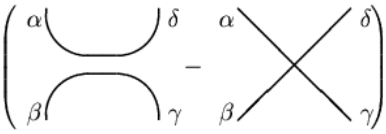
From http://www.math.toronto.edu/~drorbn/LOP.html#Weights :

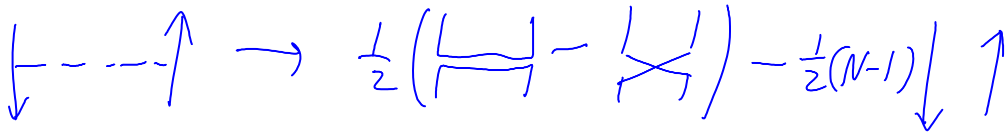
$$(21) = \frac{1}{2}(\delta_{\alpha\delta}\delta_{\beta\gamma} - \delta_{\alpha\gamma}\delta_{\beta\delta}) = \frac{1}{2}\Bigg($$

$$\Bigg).$$

The last thing to note is that

$$C_{so(N,\mathbf{C})}(k \text{ disjoint circles}) = N^k.$$

The raw $so(N)$ w.s:  $= \frac{1}{2}(N-1)$ ——— ;

so after deframing, it is



$$O^k \to N^k$$

http://katlas.math.toronto.edu/drorbn/index.php?title=AKT-09/HW2

Problem 3. The *Kauffman polynomial* $F(K)(a,z)$ (see [Kauffman]) of a knot or link K is $a^{-w(K)}L(K)$ where $w(L)$ is the writhe of K and where $L(K)$ is the regular isotopy invariant defined by the skein relations

$$L(s_\pm) = a^{\pm 1}L(s))$$

(here s is a strand and $s_\pm$ is the same strand with a $\pm$ kink added) and

$$L(\times) + L(\times) = z\left(L()\,() + L(\asymp)\right)$$

and by the initial condition $L(\bigcirc) = 1$. State and prove the relationship between F and W_{so}.

$$a^{-1}F(\overset{-}{\times}) + aF(\overset{+}{\times}) = z(F()\,() + F(\asymp))$$

also

$$(a+a^{-1})L(O^k) = z(L(O\,O^k) + L(O^k))$$

So $F(O^{k+1}) = L(O^{k+1}) = \left(\frac{a+a^{-1}}{z} - 1\right)L(O^k)$

or

$$F(O^k) = \left(\frac{a+a^{-1}}{z} - 1\right)^{k-1}$$

This is only approximate. There should also be a factor dependent on the number of comps, to somehow turn $\asymp + \times$ into $\asymp - \times$

Now set $z = i(e^{x/2} - e^{-x/2})$, $a = ie^{\frac{N-1}{2}x}$

to get

$$e^{\frac{N-1}{2}x}F(\times)-e^{-\frac{N-1}{2}x}F(\times)=\left(e^{x/2}-e^{-x/2}\right)\left(F(\,)\,(\,)+F(\asymp)\right)$$

So the w.s. satisfies:

$$\bigcirc^{K} \longrightarrow N^{K-1}$$