Exponentiation exercises mod 6T and 4T

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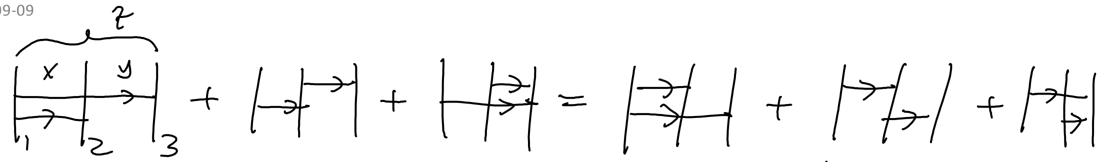
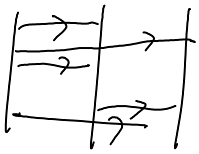
6T: 

$$[x,z]+[x,y]+[z,y]=0$$



$$[x,y]=[z,x-y]$$
$$[y,x]=[x-y,z]$$
$$=ad_z y-ad_z x$$

target: $W_1(x,z)W_2(y,z)$

$$[y,[y,x]]=[y,[x-y,z]]=[[y,x-y],z]+[x-y,[y,z]]$$
$$=[[x-y,z],z]+[x-y,[y,z]]$$

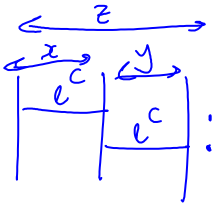
$$[x,[y,z]]=[[x,y],z]+[y,[x,z]]$$
$$=[[z,x-y],z]+$$

Can only be simplified in the world of tangles!

$$e^y e^x = e^{ad\,y}(e^x e^y) = \left(e^{ad\,y}(e^x)\right)e^y = e^{e^{ad\,y}x}\cdot e^y$$
$$=$$

$$\left(\begin{aligned}&= e^{e^y x e^{-y}} e^y\\ &= e^y e^x \cancel{e^{-y}} \cancel{e^y}\end{aligned}\right)$$

Did I ever do the corresponding exponentiation exercise mod 4T?



$$\boxed{\begin{aligned}e^y e^x &= e^{e^{ad\,y}x}\cdot e^y = e^{x+z-e^{ad\,y}z}e^y = \\ &= e^{x+y+z}e^{-e^{ad\,y}z-y}e^y = e^x e^{y+z}e^{-e^{ad\,y}z-y}e^y\end{aligned}}$$

Amazing!!!

$$[y,x]=-[y,z]$$
$$[y,[y,x]]=-[y,[y,z]] \quad \text{etc.}$$

$$e^{ad\,y}(x+y+z)=x+y+z \Rightarrow e^{ad\,y}x=x+y+z-e^{ad\,y}z-y$$

Question Can I turn "the trigonometric subset" of A_n^{hor} completely symbolic/algorithmic?

$$e^{e^y z e^{-y}+y}\, e^x = ?$$

$$x+y+z = e^{e^y z e^{-y}+y}(x+y+z)\,e^{-e^y z e^{-y}-y}$$

$$= e^{ad(e^y z e^{-y}+y)}(x+y+z)$$

$$= e^{ad(e^y z e^{-y}+y)}(x) + e^{ad(e^y z e^{-y}+y)}(e^y z e^{-y}+y)$$

$$+ e^{ad(e^y z e^{-y}+y)}(z - e^y z e^{-y})$$

$$= e^{ad(e^y z e^{-y}+y)}(x) + e^y z e^{-y}+y + \text{last term copied.}$$

So $e^{ad(e^y z e^{-y}+y)}(x) = x+z - e^y z e^{-y} + e^{ad(e^y z e^{-y}+y)}(e^y z e^{-y} - z)$

$$= x+z - e^y z e^{-y} + e^{ad(e^y z e^{-y}+y)}(e^y z e^{-y}+y)$$

$$- e^{ad(e^y z e^{-y}+y)}(y+z)$$

$$= x+z+y - e^{ad(e^y z e^{-y}+y)}(y+z)$$

So

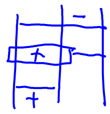
$$e^{e^y z e^{-y}+y}\, e^x = e^{ad(e^y z e^{-y}+y)}(e^x)\cdot e^{e^y z e^{-y}+y}$$

$$= e^x e^{y+z} e^{-e^{ad(e^y z e^{-y}+y)}(y+z)} \cdot e^{e^y z e^{-y}+y}$$

The hexagon @ $\Phi=1$:

$e^{y+z} \sim e^y e^z$ irrelevant.

$$e^{x+y} = e^{x+y+z-z} = e^{x+y+z}e^{-z} = e^x e^{y+z} e^{-z} =$$



A like trick in general?

$$\exp\left(\sum a_{ij} t^{ij}\right) =$$

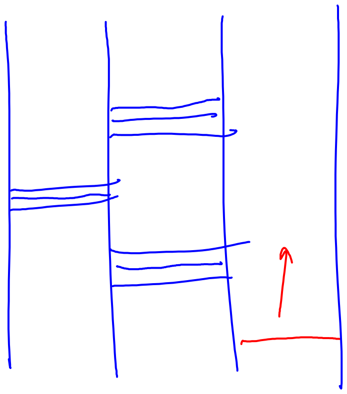
$$t^{12}+t^{23}+t^{34} = t^{12}+t^{23}+t^{34}+t^{13}+t^{24}+t^{14} - t^{13} - t^{24} - t^{14}$$

$$t^{12}+t^{23}+t^{34} = t^{12}+t^{23}+t^{34}+t^{13}+t^{24}+t^{14}-t^{13}-t^{24}-t^{14}$$

$$= t^{12}+t^{23}+t^{34}+t^{24}+t^{14}\underbrace{-t^{24}-t^{14}}$$

$$= -t^{13}+\underbrace{t^{13}+t^{12}+t^{23}+t^{34}+t^{24}+t^{14}}_{\text{Central.}}-t^{24}-t^{14}$$

$$e^{t^{12}}e^{t^{13}+t^{23}}e^{t^{14}+t^{24}+t^{34}}\cdot e^{-t^{13}-t^{24}-t^{14}}$$



$$t^{ij}\cdot x^i = [x^i, x^j]$$

$$t^{ij}\cdot x^j = [x^j, x^i]$$

$$e^{\mathrm{ad}\,x}(x+y+z) = x+y+z$$

$$e^{\mathrm{ad}\,x}y = e^{-\mathrm{ad}\,y-\mathrm{ad}\,z}y$$

$$e^{2\,\mathrm{ad}\,x}y = e^{\mathrm{ad}\,x}\left(e^{-\mathrm{ad}\,y+z}y\right) =$$

$$= e^{-\mathrm{ad}\,e^{-\mathrm{ad}\,y+z}(y+z)}e^{-\mathrm{ad}\,y+z}y = e^{-2\,\mathrm{ad}(y+z)}y$$