

Define  $\alpha, \beta: \vec{\mathcal{D}} \rightarrow \vec{\mathcal{A}}$  by

$$\alpha: \rightarrow \rightarrow \rightarrow + \leftarrow$$

$$\beta: \rightarrow \rightarrow \rightarrow - \leftarrow$$

Question What are  $\ker \alpha$  and  $\ker \beta$ ?

$\ker \alpha$  is (at least locally) 4T, by direct inspection (though a better proof would be welcome):

$$|\begin{array}{|c|} \hline \rightarrow \\ \hline \end{array}| - |\begin{array}{|c|} \hline \rightarrow \\ \hline \end{array}| = \pm (|\begin{array}{|c|} \hline \rightarrow \\ \hline \end{array}| - |\begin{array}{|c|} \hline \rightarrow \\ \hline \end{array}|) \Rightarrow 4T$$

$$|\begin{array}{|c|} \hline \rightarrow \\ \hline \end{array}| - |\begin{array}{|c|} \hline \leftarrow \\ \hline \end{array}| = \mp (\text{same})$$

(so maybe we got more than 4T, as we've used less than "all" of  $\alpha$ )

Is  $\beta(6T)$  a sum of 6TS?

$$|\begin{array}{|c|} \hline \rightarrow \\ \hline \end{array}| + |\begin{array}{|c|} \hline \rightarrow \\ \hline \end{array}| + |\begin{array}{|c|} \hline \rightarrow \\ \hline \end{array}| = |\begin{array}{|c|} \hline \rightarrow \\ \hline \end{array}| + |\begin{array}{|c|} \hline \rightarrow \\ \hline \end{array}| + |\begin{array}{|c|} \hline \rightarrow \\ \hline \end{array}|$$

$\Downarrow$

reverse both arrows ✓

reverse both arrows ✓

Reverse one arrow:

$$|\begin{array}{|c|} \hline \rightarrow \\ \hline \end{array}| + |\begin{array}{|c|} \hline \leftarrow \\ \hline \end{array}| + |\begin{array}{|c|} \hline \leftarrow \\ \hline \end{array}| \stackrel{?}{=} |\begin{array}{|c|} \hline \rightarrow \\ \hline \end{array}| + |\begin{array}{|c|} \hline \leftarrow \\ \hline \end{array}| + |\begin{array}{|c|} \hline \leftarrow \\ \hline \end{array}|$$

$$|\begin{array}{|c|} \hline \leftarrow \\ \hline \end{array}| + |\begin{array}{|c|} \hline \rightarrow \\ \hline \end{array}| + |\begin{array}{|c|} \hline \rightarrow \\ \hline \end{array}| \stackrel{?}{=} |\begin{array}{|c|} \hline \leftarrow \\ \hline \end{array}| + |\begin{array}{|c|} \hline \rightarrow \\ \hline \end{array}| + |\begin{array}{|c|} \hline \rightarrow \\ \hline \end{array}|$$

... probably not.

Some short sequences:

$$\begin{matrix} \mathcal{U} \\ \text{Sym}^d \end{matrix} \rightarrow \vec{\mathcal{D}} \xrightarrow{\alpha} \vec{\mathcal{A}} \quad \text{folds to} \quad \mathcal{A} = \vec{\mathcal{D}} / \mathcal{U}, \text{Sym}^d \mathcal{A} \xrightarrow{\alpha} \vec{\mathcal{A}}$$