

# Sabbatical Report

October-07-09  
11:31 AM

Dear Esteemed University of Toronto Colleagues,

As per University customs, here is a report of my research and scholarly accomplishments carried out during the period of my sabbatical leave, July 2008 - June 2009.

I'll start by quoting from my own paper-in-preparation, "Finite Type Invariants of  $w$ -Knotted Objects: From Alexander to Kashiwara and Vergne", <http://www.math.toronto.edu/~drorbn/papers/WKO/>:

I have a dream, at least partially founded on reality, that many of the difficult algebraic equations in mathematics, especially those that are written in graded spaces, more especially those that are related in one way or another to quantum groups, and even more especially those related to the work of Etingof and Kazhdan, can be understood, and indeed, would appear more natural, in terms of finite type invariants of various topological objects.

I believe this is the case for Drinfel'd's theory of associators, which can be interpreted as a theory of well-behaved universal finite type invariants of parenthesized tangles, and even more elegantly, as a theory of universal finite type invariants of knotted trivalent graphs.

I believe this is the case for Drinfel'd's "Grothendieck-Teichmüller group" which is better understood as a group of automorphisms of a certain algebraic structure, also related to universal finite type invariants of parenthesized tangles.

And I'm optimistic, indeed I believe, that sooner or later the work of Etingof and Kazhdan on quantization of Lie bialgebras will be re-interpreted as a construction of a well-behaved universal finite type invariant of virtual knots or of some other class of virtually knotted objects. Some steps in that direction were taken by Haviv.

I have another dream, ...

Mostly during my sabbatical I worked on realizing this (and the other) dream. The dream is long term, so I can definitely say I did not fulfil it; that would be a matter for the years to come. But I have succeeded with several special and partial cases, most noteworthy the case of " $w$ -knots", which turns out to be related to the Etingof-Kazhdan story for co-commutative Lie bialgebras or alternatively to the "Kashiwara-Vergne Conjecture". I'm in the process of writing up those results. The first 40-something pages of the paper being written are already on the web, at the same address from which the above first page was taken.

I have travelled quite a bit during my sabbatical. I gave talks at the following places:

- The Canadian Undergraduate Mathematics Conference in Toronto, "Non-Commutative Gaussian Elimination and Rubik's Cube", <http://www.math.toronto.edu/~drorbn/Talks/CUMC-0807/>.
- The Mathematical Sciences Research Institute in Berkeley, "Projectivization,  $w$ -Knots, Kashiwara-Vergne and Alekseev-Torossian", <http://www.math.toronto.edu/~drorbn/Talks/MSRI-0808/>.
- Sanbjerg, Denmark, "The Penultimate Alexander Invariant", <http://www.math.toronto.edu/~drorbn/Talks/Sandbjerg-0810/>.
- Copenhagen University Colloquium, "Algebraic Knot Theory", <http://www.math.toronto.edu/~drorbn/Talks/Copenhagen-081009/>.
- Northeastern University, Boston, "Disorganized Tidbits: Kashiwara-Vergne, Drinfel'd, Alekseev-Torossian, Alexander, Virtual knots, and Etingof-Kazhdan", <http://www.math.toronto.edu/~drorbn/Talks/Northeastern-081028/>.
- Penn State University Colloquium, "( $u$ ,  $v$ , and  $w$  knots)  $\times$  (topology, combinatorics, low algebra, and high algebra)", <http://www.math.toronto.edu/~drorbn/Talks/PSU-090205/>.
- 3 talks in Bogota, Colombia, "( $u$ ,  $v$ , and  $w$  knots)  $\times$  (topology, combinatorics, low algebra, and high algebra)", "Algebraic Knot Theory", and "Local Khovanov Homology", <http://www.math.toronto.edu/>

[~drorbn/Talks/Bogota-0902/](http://www.math.toronto.edu/~drorbn/Talks/Bogota-0902/).

- The annual "Friends of Mathematics Lecture" at Kansas State University, "(u, v, and w knots) x (topology, combinatorics, low algebra, and high algebra)" and a public lecture "The Hardest Math I've Ever Really Used", <http://www.math.toronto.edu/~drorbn/Talks/KSU-090407/>.
- 4 summer school lectures in Trieste, Italy, "(u, v, and w knots) x (topology, combinatorics, low algebra, and high algebra)" and a conference talk "Convolutions on Lie Groups and Lie Algebras and Ribbon 2-Knots", <http://www.math.toronto.edu/~drorbn/Talks/Trieste-0905/>.
- Conference talk in Paris, "Convolutions on Lie Groups and Lie Algebras and Ribbon 2-Knots", <http://www.math.toronto.edu/~drorbn/Talks/Paris-0906/>.

Also during my sabbatical I continued my work with my Ph.D. students Hernando Burgos (graduated), Jana Archibald, Peter Lee, Zsuzsanna Dancso, Louis Leong, and Karene Chu, and with my undergraduate project student Iva Halacheva.

Sincerely,



Dror Bar-Natan