

Quasi-Homomorphic R/G

October-16-09
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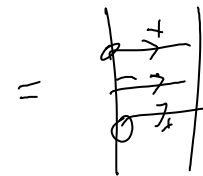
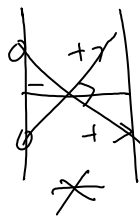
$$\begin{aligned}
 x &= \log e^x = \log(1 + (e^x - 1)) = \\
 &= (e^x - 1) - \frac{1}{2}(e^x - 1)^2 + \frac{1}{3}(e^x - 1)^3 + \dots \\
 &= \mathbb{Z} \left(\begin{array}{|c} \rightarrow \\ \hline \end{array} - \frac{1}{2} \begin{array}{|c} \rightarrow \\ \hline \rightarrow \\ \hline \end{array} + \frac{1}{3} \begin{array}{|c} \rightarrow \\ \hline \rightarrow \\ \hline \rightarrow \\ \hline \end{array} - \dots \right)
 \end{aligned}$$

$$\begin{aligned}
 e^x &\rightarrow \exp \left(\begin{array}{|c} \rightarrow \\ \hline \end{array} - \frac{1}{2} \begin{array}{|c} \rightarrow \\ \hline \rightarrow \\ \hline \end{array} + \dots \right) = \\
 &= \begin{array}{|c} | \\ \hline | \\ \hline | \\ \hline | \\ \hline \end{array} + \begin{array}{|c} \rightarrow \\ \hline \end{array} + 0 \dots
 \end{aligned}$$

$$\mathbb{Z} \left(\begin{array}{|c} \rightarrow \\ \hline \end{array} \right) = \begin{array}{|c} \rightarrow \\ \hline \rightarrow \\ \hline \end{array}$$

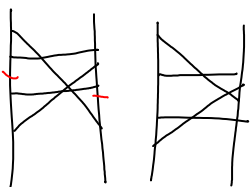


$$\mathbb{Z} \left(\begin{array}{|c} \rightarrow \\ \hline \rightarrow \\ \hline \end{array} \right) = \uparrow \uparrow$$



"simplify their exponential" maps \star to

$$\begin{aligned}
 e^{+a} - 2 + e^{-a} &= a^2 + \dots = \begin{array}{|c} \rightarrow \\ \hline \end{array} + \dots \\
 &= \begin{array}{|c} \rightarrow \\ \hline \rightarrow \\ \hline \end{array} + \dots
 \end{aligned}$$

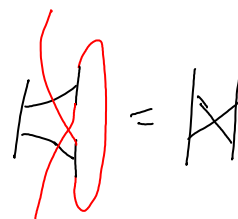
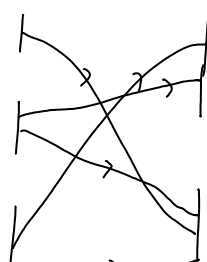


These two are not generated by H & the two multiplications.

\Rightarrow It seems that r/g is not finitely generated.
(if only 2-tangles are allowed).

Fragmented R/G (FR/G):

- Ops: 1. Disjoint union.
- 2. Glue.



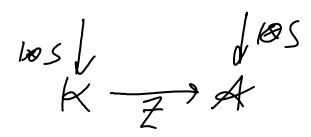
there are too many ways to glue:

there are too many ways to glue;
there ought to be a more concise presentation)



Perhaps I should start with analyzing the behavior of Z under strand reversal ("The Benjamin Button Problem")... Let's go:

Suppose $A \xrightarrow{Z} \mathbb{C}$. What's $(1 \otimes S)(a)$? $K \xrightarrow{Z} A^{\otimes a}$



$$(1 \otimes S)(a) = Z(1 \otimes S)Z^{-1}(a) = Z(1 \otimes S)(\log(1 + (A-1)))$$

$$= Z(1 \otimes S) \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} (A-1)^n = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \left[\text{diagram of } n \text{ strands with crossings} \right]$$

$$= | \text{H} | + \frac{1}{2} | \text{H} - \frac{1}{2} \text{X} | + \frac{1}{6} | \text{H} | - \frac{1}{4} (| \text{X} | + | \text{X} |) + \frac{1}{3} | \text{X} | + \dots$$

$$= | \text{H} | + \frac{1}{6} (| \text{H} | - | \text{X} |) + \dots$$