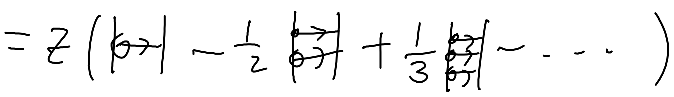
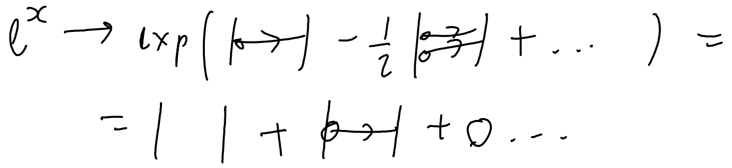
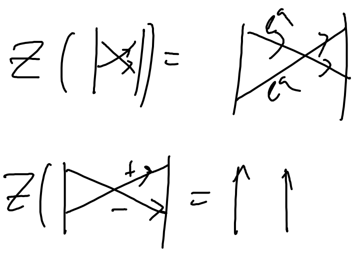
Quasi-Homomorphic R/G

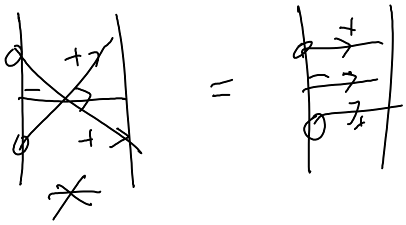
October-16-09
5:27 PM

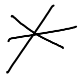
$$x = \log e^x = \log(1+(e^x-1)) =$$
$$= (e^x-1) - \frac{1}{2}(e^x-1)^2 + \frac{1}{3}(e^x-1)^3 + \dots$$

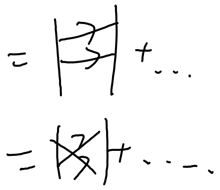


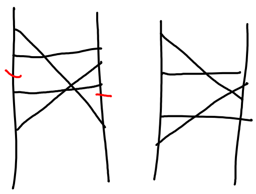






"simplify then exponential" maps  to

$e^{+a} - 2 + e^{-a} = a^2 + \dots$ 



These two are not generated by |-| & the two multiplications.

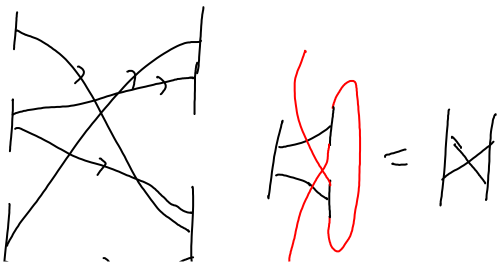
$\Rightarrow$ It seems that r/g is not finitely generated. (if only 2-tangles are allowed).

Fragmented R/G (FR/G):

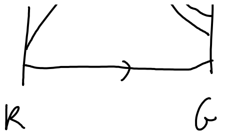
Ops: 1. Disjoint union.

2. Glue.

There are too many ways to glue:



(there are too many ways to glue; there ought to be a more concise presentation)



Perhaps I should start with analyzing the behavior of Z under strand reversal ("The Benjamin Button Problem")... Let's go:

$$\begin{array}{ccc} K & \xrightarrow{Z} & A \ni a \\ {\scriptstyle 1\otimes S}\downarrow & & \downarrow{\scriptstyle 1\otimes S} \\ K & \xrightarrow{Z} & A \end{array}$$

Suppose $A \xrightarrow{Z} e^a$. What's $(1\otimes S)(a)$?

$$(1\otimes S)(a) = Z(1\otimes S)Z^{-1}(a) = Z(1\otimes S)\left(\log(1+(A-1))\right)$$

$$= Z(1\otimes S)\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}(A-1)^n = \sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}$$