

# Hillman: Embedding 3-manifolds in 4-space

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$$\mathbb{R}P^2 \hookrightarrow \mathbb{R}^4 : (x, y, z) \sim (-x, -y, -z) \text{ w/ } x^2 + y^2 + z^2 = 1$$

$$\mapsto (x^2 - y^2, xy, yz, xz) \in \mathbb{R}^4.$$

$\beta \quad \gamma \quad \alpha \quad \beta$

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$$x^2 = \frac{\beta^2}{\alpha} \dots \text{ ok if no } x, y, z = 0.$$

If  $x = 0$ ,  $yz$  &  $y^2 + z^2 = 0$  determines  $\pm(y, z)$

If  $z = 0$ ,  $x^2 - y^2$ ,  $xy$  &  $x^2 + y^2$  determine  $\pm(x, y)$   
 $\Rightarrow$  map is 1-1.