

Consider $(1, 2, 3, \dots, n) \in \mathbb{R}^n$ and its orbit under \mathcal{S}_n .

P_n is the convex hull of that. The faces are \uparrow permutations
ordered partitions

$$[n] = S_1 \sqcup \dots \sqcup S_r \quad S_i \neq \emptyset$$

$$C_n = P_n / \mathcal{S}_r, r=2, \dots, n$$

$$QC_n = C_n \times \mathcal{S}_n / \sim$$

