BEER-2 by Peter Lee, October 19

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9:39 AM

P_n: is the poset ordered set partitions of $[n] = \{1, \ldots, n\}$.

The $(n-r)$-dim'l faces correspond to

$$n = S_1 \sqcup \ldots \sqcup S_r$$

For $2 \le r \le n$ we have S_r action on the $(n-r)$-dim'l faces

Let $C_n = P_n / \substack{\text{all} \\ S_r\text{'s}}$.

<u>Claim</u> $dvB_n \cong \pi_1(C_n)$

Consider $C_n \times S_n$. Identify $(S_1 \sqcup \ldots \sqcup S_r, \sigma)$ with $(S_1 \sqcup \ldots \sqcup S_r, \tau)$ when

$$\forall i = 1, \ldots, r, \ \forall x, y \in S_i, \ \sigma(x) < \sigma(y) \Leftrightarrow \tau(x) < \tau(y)$$

Let $QC_n = C_n \times S_n / \sim$

<u>Example</u> $(\{1,3,4\}, \{2\}, id) \sim \left(\{1,3,4\}, \{2\}, \begin{smallmatrix} 1234 \\ \downarrow \\ 1324 \end{smallmatrix}\right)$

<u>Claim</u> $\pi_1(QC_n) \cong vB_n$

There's a "split quotient":

$$dvB_n \hookrightarrow vB_n \twoheadrightarrow dvB_n$$