

```
In[*]:= pts4 = Point @@@ Permutations[Range[4]]
```

```
Out[*]=
```

```
{Point[1, 2, 3, 4], Point[1, 2, 4, 3], Point[1, 3, 2, 4], Point[1, 3, 4, 2], Point[1, 4, 2, 3],
Point[1, 4, 3, 2], Point[2, 1, 3, 4], Point[2, 1, 4, 3], Point[2, 3, 1, 4], Point[2, 3, 4, 1],
Point[2, 4, 1, 3], Point[2, 4, 3, 1], Point[3, 1, 2, 4], Point[3, 1, 4, 2], Point[3, 2, 1, 4],
Point[3, 2, 4, 1], Point[3, 4, 1, 2], Point[3, 4, 2, 1], Point[4, 1, 2, 3], Point[4, 1, 3, 2],
Point[4, 2, 1, 3], Point[4, 2, 3, 1], Point[4, 3, 1, 2], Point[4, 3, 2, 1]}
```

```
In[*]:= mat = {{1, -1, 0, 0}, {0, 1, -1, 0}, {0, 0, 1, -1}};
```

```
mat = Drop[Orthogonalize[{{1, 1, 1, 1}, {1, -1, 0, 0}, {0, 1, -1, 0}, {0, 0, 1, -1}}], 1]
```

```
Out[*]=
```

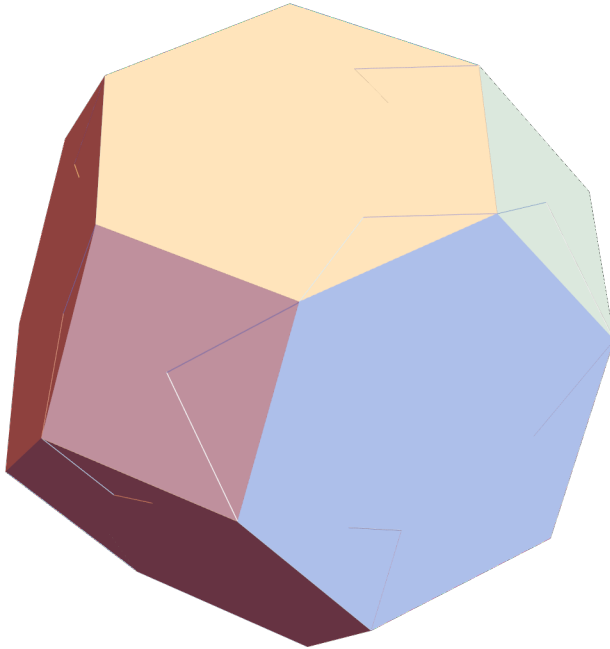
$$\left\{ \left\{ \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, -\sqrt{\frac{2}{3}}, 0 \right\}, \left\{ \frac{1}{2\sqrt{3}}, \frac{1}{2\sqrt{3}}, \frac{1}{2\sqrt{3}}, -\frac{\sqrt{3}}{2} \right\} \right\}$$

```
In[ ]:= pts3 = pts4 /. Point[xs_] => Point @@ (
  mat.{xs}
)
```

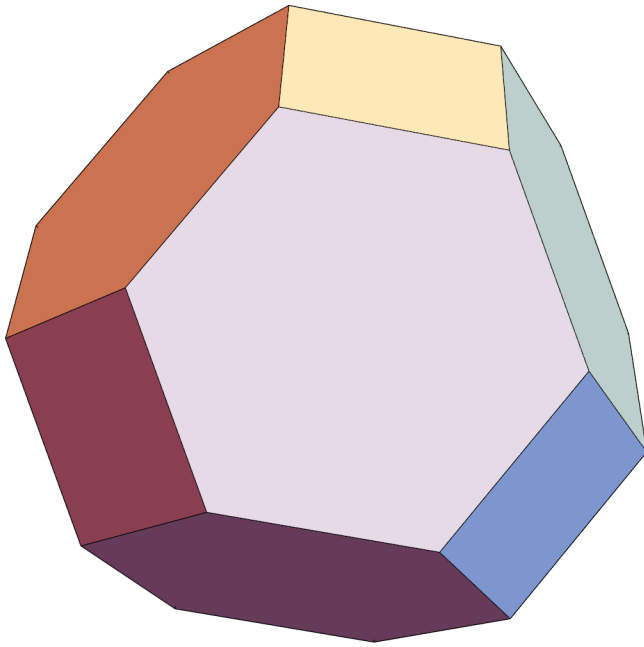
```
Out[ ]:=
```

```
{Point[ $\frac{1}{\sqrt{2}} - \sqrt{2}, \sqrt{\frac{2}{3}} + \frac{1}{\sqrt{6}} - \sqrt{6}, -\sqrt{3}$ ], Point[ $\frac{1}{\sqrt{2}} - \sqrt{2}, \frac{1}{\sqrt{6}} - \sqrt{6}, \frac{7}{2\sqrt{3}} - \frac{3\sqrt{3}}{2}$ ]},
Point[ $-\sqrt{2}, -2\sqrt{\frac{2}{3}} + \sqrt{\frac{3}{2}} + \frac{1}{\sqrt{6}}, -\sqrt{3}$ ], Point[ $-\sqrt{2}, -4\sqrt{\frac{2}{3}} + \sqrt{\frac{3}{2}} + \frac{1}{\sqrt{6}}, \frac{5}{2\sqrt{3}} - \frac{\sqrt{3}}{2}$ ]},
Point[ $\frac{1}{\sqrt{2}} - 2\sqrt{2}, \frac{1}{\sqrt{6}}, \frac{7}{2\sqrt{3}} - \frac{3\sqrt{3}}{2}$ ], Point[ $\frac{1}{\sqrt{2}} - 2\sqrt{2}, 2\sqrt{\frac{2}{3}} + \frac{1}{\sqrt{6}} - \sqrt{6}, \frac{5}{2\sqrt{3}} - \frac{\sqrt{3}}{2}$ ]},
Point[ $-\frac{1}{\sqrt{2}} + \sqrt{2}, \sqrt{\frac{2}{3}} + \frac{1}{\sqrt{6}} - \sqrt{6}, -\sqrt{3}$ ], Point[ $-\frac{1}{\sqrt{2}} + \sqrt{2}, \frac{1}{\sqrt{6}} - \sqrt{6}, \frac{7}{2\sqrt{3}} - \frac{3\sqrt{3}}{2}$ ]},
Point[ $-\frac{3}{\sqrt{2}} + \sqrt{2}, \sqrt{\frac{3}{2}}, -\sqrt{3}$ ], Point[ $-\frac{3}{\sqrt{2}} + \sqrt{2}, \sqrt{\frac{3}{2}} - \sqrt{6}, \sqrt{3}$ ]},
Point[ $-\sqrt{2}, 2\sqrt{\frac{2}{3}}, \frac{7}{2\sqrt{3}} - \frac{3\sqrt{3}}{2}$ ], Point[ $-\sqrt{2}, 0, \sqrt{3}$ ]},
Point[ $\sqrt{2}, -2\sqrt{\frac{2}{3}} + \sqrt{\frac{3}{2}} + \frac{1}{\sqrt{6}}, -\sqrt{3}$ ], Point[ $\sqrt{2}, -4\sqrt{\frac{2}{3}} + \sqrt{\frac{3}{2}} + \frac{1}{\sqrt{6}}, \frac{5}{2\sqrt{3}} - \frac{\sqrt{3}}{2}$ ]},
Point[ $\frac{3}{\sqrt{2}} - \sqrt{2}, \sqrt{\frac{3}{2}}, -\sqrt{3}$ ], Point[ $\frac{3}{\sqrt{2}} - \sqrt{2}, \sqrt{\frac{3}{2}} - \sqrt{6}, \sqrt{3}$ ]},
Point[ $\frac{3}{\sqrt{2}} - 2\sqrt{2}, \sqrt{\frac{2}{3}} + \sqrt{\frac{3}{2}}, \frac{5}{2\sqrt{3}} - \frac{\sqrt{3}}{2}$ ], Point[ $\frac{3}{\sqrt{2}} - 2\sqrt{2}, \sqrt{\frac{3}{2}}, \sqrt{3}$ ]},
Point[ $-\frac{1}{\sqrt{2}} + 2\sqrt{2}, \frac{1}{\sqrt{6}}, \frac{7}{2\sqrt{3}} - \frac{3\sqrt{3}}{2}$ ], Point[ $-\frac{1}{\sqrt{2}} + 2\sqrt{2}, 2\sqrt{\frac{2}{3}} + \frac{1}{\sqrt{6}} - \sqrt{6}, \frac{5}{2\sqrt{3}} - \frac{\sqrt{3}}{2}$ ]},
Point[ $\sqrt{2}, 2\sqrt{\frac{2}{3}}, \frac{7}{2\sqrt{3}} - \frac{3\sqrt{3}}{2}$ ], Point[ $\sqrt{2}, 0, \sqrt{3}$ ]},
Point[ $-\frac{3}{\sqrt{2}} + 2\sqrt{2}, \sqrt{\frac{2}{3}} + \sqrt{\frac{3}{2}}, \frac{5}{2\sqrt{3}} - \frac{\sqrt{3}}{2}$ ], Point[ $-\frac{3}{\sqrt{2}} + 2\sqrt{2}, \sqrt{\frac{3}{2}}, \sqrt{3}$ ]}
```

```
In[ ]:= l = Length[pts3];  
Graphics3D[{EdgeForm[None],  
  Table[  
    Polygon[List @@@ pts3[[{i, j, k}]],  
    {i, 1, l-2}, {j, i+1, l-1}, {k, j+1, l}  
  ]},  
  Boxed → False  
]  
Out[ ]=
```



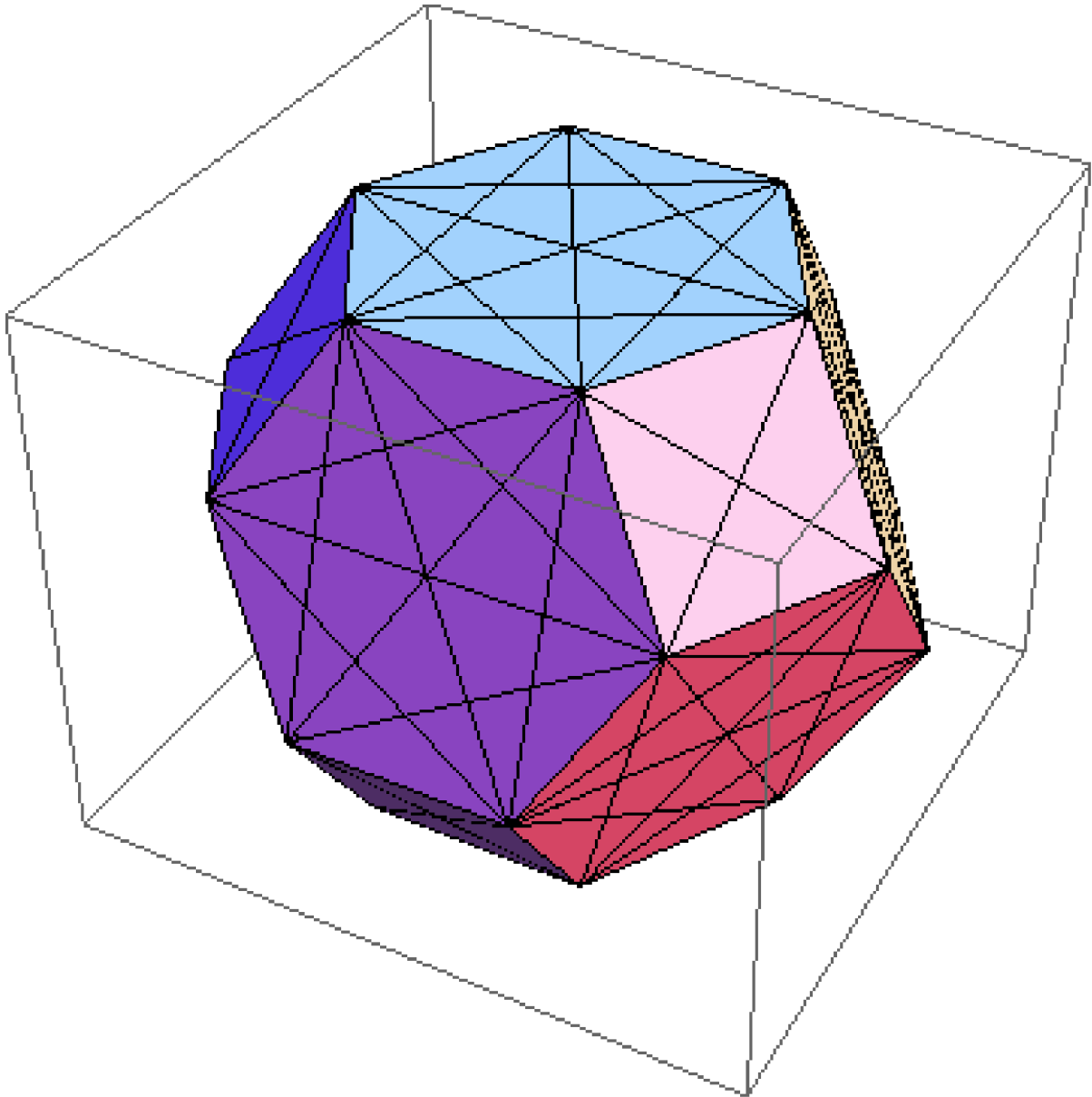
```
In[*]:= Graphics3D[ConvexHullMesh[pts3 /. Point -> List], Boxed -> False]  
Out[*]=
```



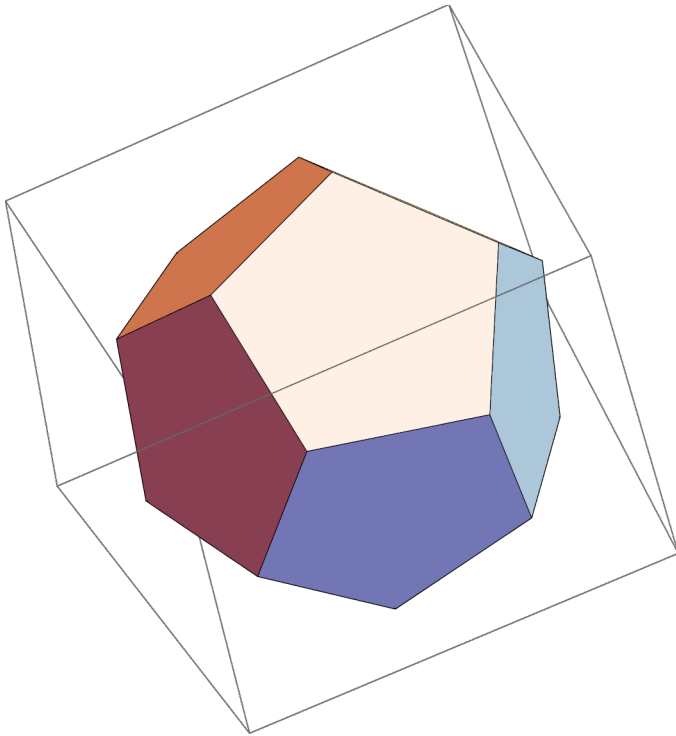
```

l = Length[pts3];
Rasterize[Graphics3D[
  Table[
    Polygon[List @@ pts3[{i, j, k}],
      {i, 1, l-2}, {j, i+1, l-1}, {k, j+1, l}
    ]
  ]
]]

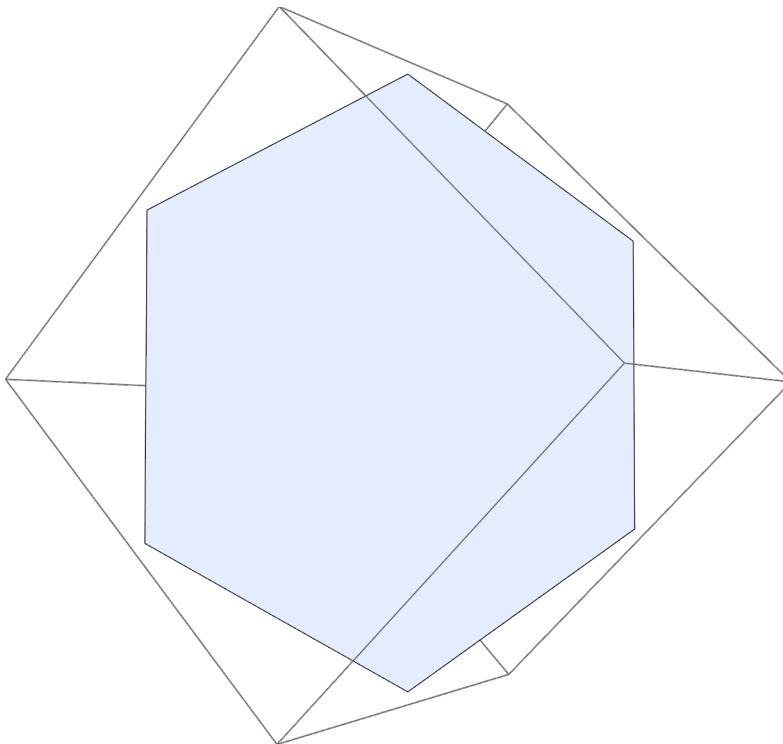
```



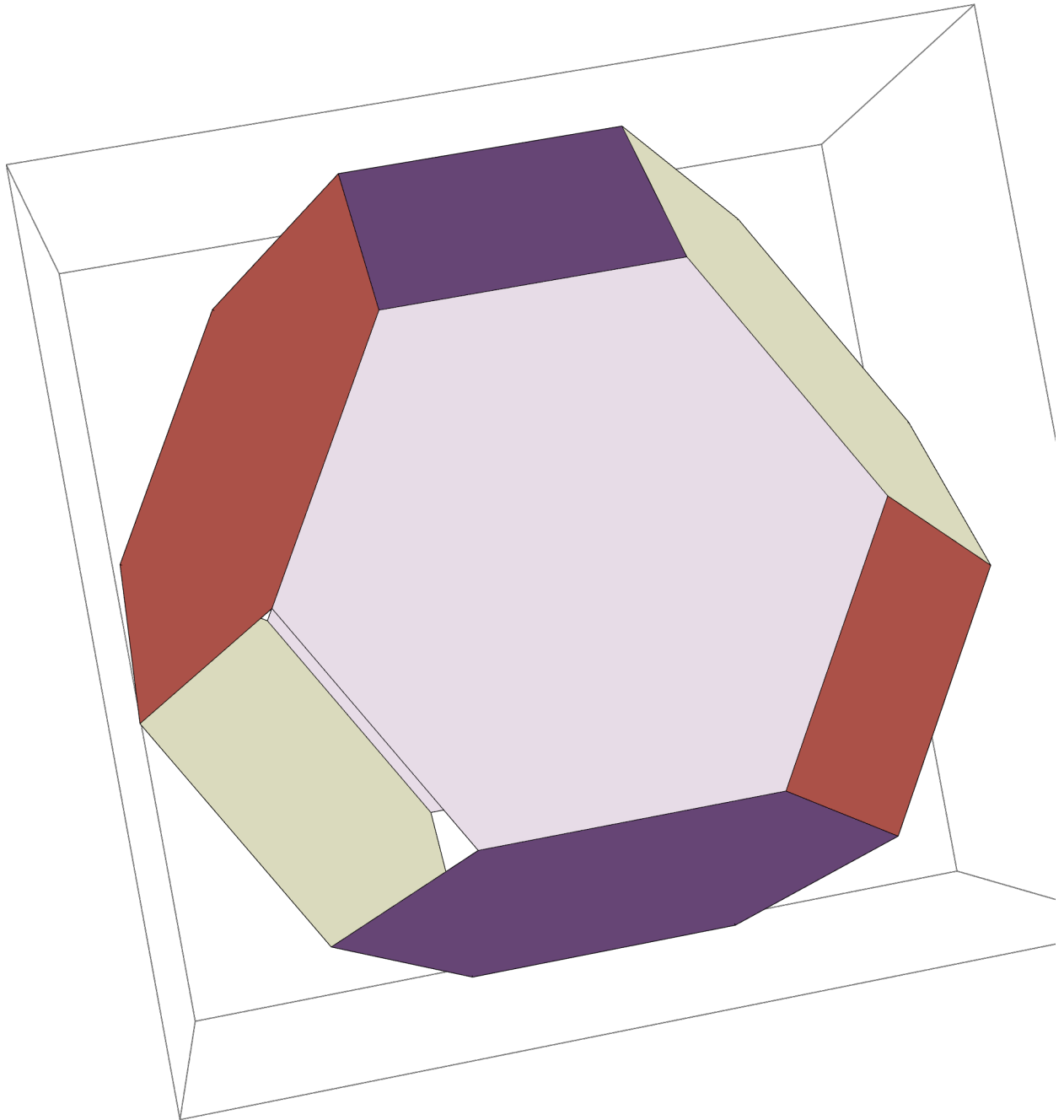
`PolyhedronData["Dodecahedron"]`



```
bhex = {{1, 2, 3}, {1, 3, 2}, {2, 3, 1}, {3, 2, 1}, {3, 1, 2}, {2, 1, 3}};  
Graphics3D[Polygon[bhex]]
```



```
Graphics3D[
  Join[
    Table[
      Polygon[Insert[#, 4, i] & /@ bhex],
      {i, 4}
    ],
    Table[
      Polygon[Insert[#, 1, i] & /@ (1 + bhex)],
      {i, 4}
    ]
  ] /. {x1_, x2_, x3_, x4_} => mat.{x1, x2, x3, x4}
]
```




```

Graphics3D[{Opacity[0.7],
  Join[
    Table[
      Polygon[Insert[#, 4, i] & /@ bhex],
      {i, 4}
    ],
    Table[
      Polygon[Insert[#, 1, i] & /@ (1 + bhex)],
      {i, 4}
    ],
    {
      Polygon[{{1, 2, 3, 4}, {1, 2, 4, 3}, {2, 1, 4, 3}, {2, 1, 3, 4}}],
      Polygon[{{1, 3, 2, 4}, {1, 4, 2, 3}, {2, 4, 1, 3}, {2, 3, 1, 4}}],
      Polygon[{{1, 4, 3, 2}, {1, 3, 4, 2}, {2, 3, 4, 1}, {2, 4, 3, 1}}],
      Polygon[{{3, 1, 2, 4}, {4, 1, 2, 3}, {4, 2, 1, 3}, {3, 2, 1, 4}}],
      Polygon[{{4, 2, 3, 1}, {3, 2, 4, 1}, {3, 1, 4, 2}, {4, 1, 3, 2}}],
      Polygon[{{3, 4, 1, 2}, {4, 3, 1, 2}, {4, 3, 2, 1}, {3, 4, 2, 1}}]
    }
  ] /. {x1_Integer, x2_Integer, x3_Integer, x4_Integer} => mat.{x1, x2, x3, x4}
}
]

```

