

# Expansions for semi-direct products

September-08-09  
7:49 PM

$$\begin{array}{ccccccc}
 1 & \longrightarrow & F & \overset{\longleftarrow}{\longleftarrow} & G & \overset{\longleftarrow}{\longleftarrow} & H \longrightarrow 1 \\
 & & \downarrow & & & & \downarrow \\
 0 & \longrightarrow & \text{gr}F & \longrightarrow & \text{gr}G & \overset{\longleftarrow}{\longleftarrow} & \text{gr}H \longrightarrow 0
 \end{array}$$

||| |

Can we at least rescue a theorem of the form "here's  $A(G)$  in terms of  $A(H)$  &  $A(F)$ " out of this?

$$abc = cba$$

$$[a, b] = aba^{-1}b^{-1}$$

$$\begin{aligned}
 abc a^{-1} b^{-1} c^{-1} &= abcc^{-1} a^{-1} b^{-1} [b, a, c] \\
 &= [a, b] \cdot [b, [a, c]] [a, c] [b, c]
 \end{aligned}$$

$$\begin{aligned}
 [a, b, c] &= abc b^{-1} a^{-1} c^{-1} = a [b, c] c a^{-1} c^{-1} \\
 &= [a, [b, c]] [b, c] a c a^{-1} c^{-1} \\
 &= [a, [b, c]] [b, c] [a, c]
 \end{aligned}$$

$$aba = bab$$