Pensieve Header: Figure out if there is a homomorphic expansion for regular red-over-green tangles. (only a start...)

```
NormalizeP[P[p___]] := P[p] /. Thread[Sort[{p}] → Range[Length[{p}]]];
Basis[n_] := DeleteCases[
   Permutations[P @@ Range[n]],
   Alternatives[
    P[___, i_, j_, ___] /; i == j + 1 ,
    P[___, i_, j_, k_, ___] /; i == j + 2 && k == j + 1
   ]
  ];
P /: p1_P ** p2_P := Join[p1, (Length[p1] + #) & /@ p2];
ASeries /: c_?NumberQ * a_ASeries := Expand[c #] & /@ a;
ASeries /: a1_ASeries + a2_ASeries := Module[
   {m = Min[Length[a1], Length[a2]]},
   ASeries @@ (Take[List @@ a1, m] + Take[List @@ a2, m])
  ];
ASeries /: a1_ASeries ** a2_ASeries := Module[
   {m = Min[Length[a1], Length[a2]] - 1, P1, P2},
   ASeries @@ Table[
      Sum[
       Expand[(a1[[d1 + 1]] /. P → P1) * (a2[[d - d1 + 1]] /. P → P2)],
       {d1, 0, d}
      ],
      {d, 0, m}
     ] /. p1_P1 * p2_P2 :> (P @@ p1) ** (P @@ p2)
  ];
Invert[ASeries[P[], x___]] := Module[
   {s, t, k},
   t = ReplacePart[ASeries @@ Table[0, {1 + Length[{x}]}], 1 → P[]];
   t + Sum[
     t = (-t) ** ASeries[0, x],
     {Length[{x}]}
    ]
  ];
InsertAt[k_Integer, P[p1___], P[p2___]] :=
  NormalizeP[P @@ Flatten[{p2} /. k → k + {p2} / 100]];
GT[p_P] := Reverse[p] //. {
    P[l___, i_, j_, r___] /; i == j + 1 :> P[l, j, i, r],
    P[l___, i_, j_, k_, r___] /; i == j + 2 && k == j + 1 :> P[l, j, k, i, r]
   };
GT[expr_] := expr /. p_P :> GT[p];
EQ1[R_ASeries] := R ** Invert[R];
EQ2[R_ASeries] := GT[R] ** GT[Invert[R]];
EQ[R_ASeries] := {EQ1[R], EQ2[R]};
```

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InsertAt[2, P[3, 2, 1], P[3, 1, 2]]
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P[5, 1, 4, 2, 3]
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