Kirk in Bonn

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8:13 AM

1. Homology Cobordisms of Hommology 3-spheres.
2. Knot Concordance.

} both are "3+10"

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1. M^3, $\partial M = \phi$, $H_1(M) = 0$ } H - 3-sphere.

$\Theta^3_H = \left\langle \begin{matrix}\text{Homology} \\ \text{3-spheres}\end{matrix}, + = \begin{matrix}\text{Connected} \\ \text{sum}\end{matrix}, \begin{matrix}\Sigma \sim 0 \text{ if} \\ \Sigma = \partial X \text{ with} \\ H(X) = H(\text{ball})\end{matrix} \right\rangle$

example $\frac{1}{2}$ surgery on Y_1 is a non-trivial H-sphere but it is trivial in Θ^3_H.

example The Poincare sphere $\Sigma(2,3,5)$ is non-trivial (has infinite order (using classical CS)) in Θ^3_H.

Matumoto / Galewski-Stern: Every compact closed topological manifold of dim ≥ 5 can be triangulated iff $\exists$ a homology 3-sphere Σ w/ odd casson invariant but with $\Sigma \# \Sigma = 0$ in Θ^3_H

Fintushel-Stern / Furuto: $\{\Sigma(2,3,6k-1) \mid k \in \mathbb{Z}\}$ are linearly independent in Θ^3_H.

Knot concordance The group $\mathcal{C}$ generated by all knots (& connected sum) mod $K \sim 0$ if

$\exists \left(\begin{matrix}\text{smoothly embedded} \\ \text{topologically loc. flat}\end{matrix} \right) D^2 \subset B^4$ s.t $\partial D^2 = K$.

{Kirk's absolutely most favorite theorem in CS-land aside (is Rasmussen's construction (based on Khovanov homology) of a homomorphism $\mathcal{C} \to \mathbb{Z}$

Rudolph's conjecture Algebraic knots are lin. indep. in $\mathcal{C}$.

Let K_{pqrs} be the (rs) cable of the (pq) torus knot. Then

Thm $\{K_{237p} : p \text{ prime}\}$ are lin. indep.

Aside: the only knot w/ up to 12 xings whose triviality in $\mathcal{C}$ is not settled is K12a631