Beasley in Bonn: Localization for Wilson Loops in CS Theory

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8:10 AM

$$Z(k, C, R) := \frac{1}{\mathrm{Vol}(\mathcal{G})} \int_{\mathcal{A}} \mathcal{D}A \cdot W_R(C) \exp\left(i \frac{k}{4\pi} CS(A)\right)$$

on M, compact 3-manifold & given G, compact Lie group

Non Abelian Localization

$$S = \frac{1}{2}(\mu, \mu)$$

$$Z(\epsilon) = \frac{1}{\mathrm{Vol}(H)} \int_X \exp\left(\Omega - \frac{1}{2\epsilon}(\mu, \mu)\right)$$

where (X, Ω) is a symplectic manifold with a Hamiltonian action of a Lie gp H

$\mu : X \to \mathfrak{h}^*$ The moment map.

$(\cdot, \cdot)$ inv. form on $\mathfrak{h}/\mathfrak{h}^*$

Example 2d YM on (Σ, ω) (ω a symplectic form on a surface Σ)

$X = \mathcal{A}$, $\Omega = -\int_\Sigma \mathrm{Tr}(\delta A \wedge \delta A)$

H = Gauge transformations.

μ turns out to be F (2-forms are dual to 0-forms)

$(\phi, \phi) := -\int_\Sigma \mathrm{Tr}(\phi \wedge * \phi)$

Symplectic Geometry of CS theory.

κ section of Ω^1_M, $\kappa \wedge d\kappa \neq 0$ everywhere.

If M's a Seifert manifold, choose

"shift-symmetry" S: $\delta A = \kappa \sigma$, σ on arbitrary sect. of $\Omega^1_M \otimes \mathfrak{g}$; $\delta\Phi = \sigma$.

$$S(A,\Phi) = CS(A - \kappa\Phi)$$
$$= CS(A) - \int_M [2\kappa \wedge Tr(\Phi F_A - \kappa \wedge d\kappa Tr(\Phi^2)]$$

$$Z(\kappa) = \int \mathcal{D}A \mathcal{D}\Phi \exp \exp \frac{ik}{4\pi} S(A,\Phi)$$

1. Fix $\Phi = 0 \Rightarrow \tilde{Z} = Z$
2. Integrate out $\Phi \Rightarrow$

$$S(A) = CS(A) - \int \frac{1}{\kappa \wedge d\kappa} Tr[(\kappa \wedge F_A)^2]$$

still shift invariant!

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symplectic data: $X = \mathcal{A}/\mathcal{G}$, $\Omega = \int_M \kappa \wedge Tr(\delta A \wedge \delta A)$

$\mathcal{G}_0 =$ Gauge transformations (identity component), Centrally extended:

$$U(1) \hookrightarrow \tilde{\mathcal{G}}_0 \longrightarrow \mathcal{G}_0$$

$H: U(1)_R \ltimes \tilde{\mathcal{G}}_0$ $\quad \mu, \quad S(A) = \frac{1}{2}(\mu, \mu)$

$\uparrow$ rotations of M

Adding The Wilson Loop:

$$W_R(C) = Tr_R P\exp\left(-\oint_C A\right)$$

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