

- What property of an algebraic structure allows for "a Polyak algebra" for that structure?
- Re-think the presentation of the topic of Polyak algebras.
- The details of "Red over Green Tangles".
- Is there a homomorphic expansion for r/g-tangles?

Projectivization of a group G :

$$G = \langle g_1, \dots, g_k \mid R_1, \dots, R_\ell \rangle \quad P = (g_1, \dots, g_k \mid R_1, \dots, R_\ell)$$

$$G/I^{n+1} \cong \left\langle \bar{g}_i, \bar{g}_i^{-1} \mid \begin{array}{l} \bar{g}_i \bar{g}_{-i} + \bar{g}_i + \bar{g}_{-i} = 0 \\ R_j // (g_i \rightarrow \bar{g}_i + 1, g_i^{-1} \rightarrow \bar{g}_i^{-1} + 1) \end{array} \right\rangle \begin{array}{l} \text{Words} \\ \text{of} \\ \text{length} \\ \geq n \\ J_{n+1} \end{array}$$

by $w \mapsto w/s$

$$(g_i^{-1}, g_i^{-1}) \longleftarrow (\bar{g}_i, \bar{g}_i^{-1}) \left\{ \begin{array}{l} \text{key to verification:} \\ g_i g_i^{-1} = (\bar{g}_i + 1)(\bar{g}_i^{-1} + 1) - 1 = \bar{g}_i \bar{g}_i^{-1} + \bar{g}_i + \bar{g}_i^{-1} \end{array} \right.$$

$$\cong \left\langle \bar{g}_i \mid R_j // \left(\begin{array}{l} g_i \rightarrow \bar{g}_i + 1 \\ g_i^{-1} \rightarrow 1 - \bar{g}_i + \bar{g}_i^{-2} \dots \end{array} \right) \right\rangle / J_{n+1}$$

$$=: \bar{G} / J_{n+1}$$

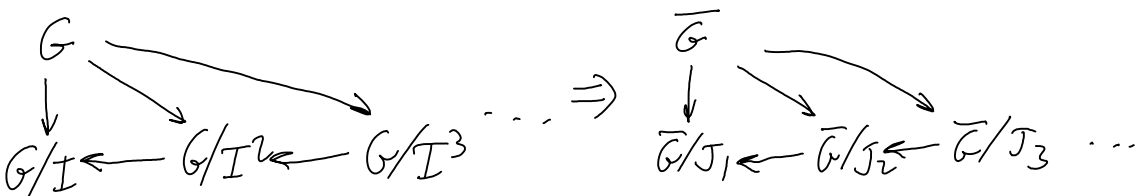
Replacing 1 with λ fails:

failure of 2nd isomor. $\left\{ \begin{array}{l} (\bar{g} + \lambda)(\bar{g}_- + \lambda) = 1 \Rightarrow \bar{g} \bar{g}_- + \bar{g} \lambda + \lambda \bar{g}_- + \lambda^2 = 1 \\ \Rightarrow \bar{g}_- = \lambda^{-1} (1 - \lambda^2 - \bar{g} \lambda - \bar{g} \bar{g}_-) \end{array} \right.$ *does not lead to a convergent power series.*

Failure of key point:

$$\bar{g}_1 \bar{g}_2 = (g_1 g_2 - \lambda) = (\bar{g}_1 + \lambda)(\bar{g}_2 + \lambda) - \lambda = \underbrace{\bar{g}_1 \bar{g}_2}_{\text{good}} + \underbrace{\bar{g}_1 \lambda + \lambda \bar{g}_2}_{\text{not so good}} + \underbrace{\lambda^2}_{\text{bad}}$$

But everything does work for "Algebraic structures with Identity", assuming the set of generators is "invariant under (all) multiplications by (all) identities".



$$\begin{array}{c}
 G \\
 \swarrow \quad \searrow \\
 G/I \leftarrow G/I^2 \leftarrow G/I^3 \dots \Rightarrow \bar{G} \\
 \downarrow \quad \swarrow \quad \searrow \\
 G/I \leftarrow G/I^2 \leftarrow G/I^3 \dots \Rightarrow \bar{G} \\
 \downarrow \quad \swarrow \quad \searrow \\
 \bar{G}/J_1 \leftarrow \bar{G}/J_2 \leftarrow \bar{G}/J_3 \dots
 \end{array}$$

$$I^n/I^{n+1} = \ker(G/I^{n+1} \rightarrow G/I^n) = \ker(\bar{G}/J_{n+1} \rightarrow \bar{G}/J_n) =: A_n$$

Let

$$0 \rightarrow \mathcal{R}_n \rightarrow J_n/J_{n+1} \rightarrow \bar{G}/J_{n+1} \rightarrow \bar{G}/J_n \rightarrow 0$$

Then

$$A_n = (J_n/J_{n+1})/\mathcal{R}_n = \left(\begin{array}{c} \text{n-letter words} \\ \text{in } \bar{g}_1 \dots \bar{g}_k \end{array} \right) / \mathcal{R}_n$$

Let

$$\mathcal{R}_n^P := \left\{ \begin{array}{l} \text{n-saturations of} \\ \text{least terms in } \mathcal{R}_n // S \end{array} \right\} \subset \mathcal{R}_n$$

Question Does $\mathcal{R}_n^P = \mathcal{R}_n$?