August 26, 2009

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7:16 AM

- What property of an algebraic structure allows for "a Polyak algebra" for that structure?
- Re-think the presentation of the topic of Polyak algebras.
- The details of "Red over Green Tangles".
- Is there a homomorphic expansion for r/g-tangles?

Projectivization of a group G:

$$G=\langle g_1\dots g_k \mid R_1\dots R_l\rangle \qquad P=(g_1\dots g_k \mid R_1\dots R_l)$$

$$G/I^{n+1} \cong \left\langle \bar g_i, \bar g_{-i} \;\middle|\; \begin{array}{l} \bar g_i\bar g_{-i}+\bar g_i+\bar g_{-i}=0 \\ R_j /\!/ \underbrace{(g_i\to\bar g_i+1,\ g_i^{-1}\to \bar g_{-i}+1)}_{S \text{ (for "substitution rule")}} \end{array} \right\rangle \Big/ \underbrace{\text{Words of length} \geq n}_{J_{n+1}}$$

by

$$W \mapsto W/\!/S$$
$$(g_i-1,\ g_i^{-1}-1) \longleftarrow\!\shortmid\ (\bar g_i,\ \bar g_{-i})$$

Key to verification: $(g_1g_2-1)=(\bar g_1+1)(\bar g_2+1)-1=\bar g_1\bar g_2+\bar g_1+\bar g_2$

$$\cong \left\langle \bar g_i \;\middle|\; R_j /\!/ \left(\begin{array}{l} g_i\to\bar g_i+1 \\ g_i^{-1}\to 1-\bar g_i+\bar g_i^2-\dots \end{array}\right) \right\rangle \Big/ J_{n+1}$$

$$=: \bar G / J_{n+1}$$

Replacing 1 with λ fails:

failure of 2nd isomor.:

$$(\bar g+\lambda)(\bar g_-+\lambda)=1 \Rightarrow \bar g\bar g_- + \bar g\lambda+\lambda\bar g_-+\lambda^2=1$$
$$\Rightarrow \bar g_- = \lambda^{-1}(1-\lambda^2-\bar g\lambda-\bar g\bar g_-)$$

does not lead to a convergent power series.

Failure of key point:

$$\overline{g_1g_2}=(g_1g_2-\lambda)=(\bar g_1+\lambda)(\bar g_2+\lambda)-\lambda=\underbrace{\bar g_1\bar g_2}_{\text{good}}+\underbrace{\bar g_1\lambda+\lambda\bar g_2}_{\text{not so good}}+\underbrace{(\lambda^2-\lambda)}_{\text{bad}}$$

But everything does work for "Algebraic Structures with Identity", assuming the set of generators is "invariant under (all) multiplications by (all) identities".

$$\begin{tikzcd}
G \arrow[d] \arrow[dr] \arrow[drr] & & & \\
G/I & G/I^2 \arrow[l] & G/I^3 \arrow[l] & \dots
\end{tikzcd}
\Rightarrow
\begin{tikzcd}
\bar G \arrow[d] \arrow[dr] \arrow[drr] & & & \\
\bar G/J_1 & \bar G/J_2 \arrow[l] & \bar G/J_3 \arrow[l] & \dots
\end{tikzcd}$$

$$\begin{tikzcd} G \arrow[d] \arrow[dr] \arrow[drr, bend left] & & \\ G/I & G/I^2 \arrow[l] & G/I^3 \arrow[l] \; \cdots \end{tikzcd} \Rightarrow \begin{tikzcd} \bar{G} \arrow[d] \arrow[dr] \arrow[drr, bend left] & & \\ \bar{G}/J_1 & \bar{G}/J_2 \arrow[l] & \bar{G}/J_3 \arrow[l] \; \cdots \end{tikzcd}$$

$$I^n/I^{n+1} = \ker\left(G/I^{n+1} \to G/I^n\right) = \ker\left(\bar{G}/J_{n+1} \to \bar{G}/J_n\right) =: A_n$$

Let
$$0 \to R_n \to J_n/J_{n+1} \to \bar{G}/J_{n+1} \to \bar{G}/J_n \to 0$$

Then
$$A_n = \left(J_n/J_{n+1}\right)/R_n = \begin{pmatrix} n\text{-letter words} \\ \text{in } \bar{g}_1 \cdots \bar{g}_k \end{pmatrix} / R_n$$

Let
$$R_n^p := \begin{Bmatrix} n\text{-saturations of} \\ \text{least terms in } R_j//S \end{Bmatrix} \subset R_n$$

<u>Question</u> Does $R_n^p = R_n$?