```mathematica
Dim[QuotientSpace[diags_List, rels_List], opts___] := Module[
   {
    debug = Debug /. {opts} /. Debug → False,
    diagtoindex, mat, rel
   },
   diagtoindex = Dispatch[Thread[Rule[diags, Range[Length[diags]]]]];
   mat = SparseArray[
     Join @@ Table[
       rel = rels[[i]];
       {i, # /. diagtoindex} → Coefficient[rel, #] & /@ Cases[{rel}, diag_P, Infinity],
       {i, Length[rels]}
      ],
     {Length[rels], Length[diags]}
    ];
   If[debug, Print[Date[], ": Computed mat..."]];
   Length[diags] - MatrixRank[mat]
  ];
NormalizeP[P[p___]] := P[p] /. Thread[Sort[{p}] → Range[Length[{p}]]];
Ps[n_] := Permutations[P @@ Range[n]];
Rs[k_, n_] := Module[
   {j, q},
   Join @@ Table[
     Sum[
        (# /. j → P @@ (j + 0.01 * Range[q]))
         - (# /. j → P @@ (j - 0.01 * Range[q])),
        {q, 2, n - k + 1}
       ] & /@ Ps[k],
      {j, k}
     ] /. p_P ⧴ NormalizeP[Flatten[p]]
  ];
ASpace[n_] := QuotientSpace[Ps[n], Rs[n - 1, n]];
PSpace[n_] := QuotientSpace[
   Flatten[Ps /@ Range[n]],
   Flatten[Rs[#, n] & /@ Range[n - 1]]
  ];
DimA[m_, opts___] := Dim[ASpace[m, opts], opts];
DimP[m_, opts___] := Dim[PSpace[m, opts], opts];

DimA /@ {2, 3, 4, 5, 6, 7}

{1, 2, 8, 42, 258, 1824}

Length /@ ASpace[7]

QuotientSpace[5040, 4320]

5040 - 4320

720

DimP /@ {2, 3, 4, 5, 6, 7}

{2, 4, 12, 54, 312, 2136}
```

```
      Length /@ PSpace[7]

      QuotientSpace[5913, 5039]

      5913 - 5039

      874
```

In[7]:=
```
      Basis[n_] := DeleteCases[
         Permutations[P @@ Range[n]],
         Alternatives[
          P[___, i_, j_, ___] /; i == j + 1,
          P[___, i_, j_, k_, ___] /; i == j + 2 && k == j + 1
         ]
        ]
```

In[8]:=
```
      Basis[4]
```

Out[8]=
```
      {P[1, 2, 3, 4], P[1, 3, 4, 2], P[2, 3, 1, 4], P[2, 3, 4, 1],
       P[2, 4, 1, 3], P[3, 1, 4, 2], P[3, 4, 1, 2], P[4, 1, 2, 3]}
```

In[12]:=
```
      Length /@ Basis /@ Range[10]
```

Out[12]=
```
      {1, 1, 2, 8, 42, 258, 1824, 14 664, 132 360, 1 326 120}
```

http://www.research.att.com/~njas/sequences/A013999 says:

In[15]:=
```
      Sum[n! x^n (1 - x) ^n + O[x] ^11, {n, 0, 10}]
```

Out[15]= $1 + x + x^2 + 2 x^3 + 8 x^4 + 42 x^5 + 258 x^6 + 1824 x^7 + 14 664 x^8 + 132 360 x^9 + 1 326 120 x^{10} + O[x]^{11}$

In[46]:=
```
      PrimitivesToAll[p_List] := Module[{x},
         CoefficientList[Series[
           Product[(1 - x^i) ^ (- p[[i]]), {i, Length[p]}],
           {x, 0, Length[p]}], x]
        ];
      PrimitivesToAll[{1, 0, 1, 6, 34, 215, 1560, 12 785, 117 276, 1 190 309}] // Rest
```

Out[47]=
```
      {1, 1, 2, 8, 42, 258, 1824, 14 664, 132 360, 1 326 120}
```