Non Commutative Gaussian Elimination - Program 3

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Amended from a similar notebook by Dror Bar-Natan and Itai Bar-Natan. The original version is at http://www.math.toronto.edu/~drorbn/Misc/SchreierSimsRubik/.

Pensieve Header: NCGE Program 3 - replacing tricks with better ones when possible, recursively. The results are pathetic.

The Cube

The Generating Permutations

```
n = 54; $RecursionLimit = 2^16;
Generators = {
   M[{18, 27, 36, 4, 5, 6, 7, 8, 9, 3, 11, 12, 13, 14, 15, 16, 17,
     45, 2, 20, 21, 22, 23, 24, 25, 26, 44, 1, 29, 30, 31, 32, 33, 34, 35, 43,
     37, 38, 39, 40, 41, 42, 10, 19, 28, 52, 49, 46, 53, 50, 47, 54, 51, 48},
    {BottomFace}, 1],
   M[{1, 2, 3, 4, 5, 6, 16, 25, 34, 10, 11, 9, 15, 24, 33, 39, 17,
     18, 19, 20, 8, 14, 23, 32, 38, 26, 27, 28, 29, 7, 13, 22, 31, 37, 35, 36,
     12, 21, 30, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54},
    {TopFace}, 1],
   M[{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17,
     18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 31, 32, 33, 34, 35, 36, 48, 47, 46,
     39, 42, 45, 38, 41, 44, 37, 40, 43, 30, 29, 28, 49, 50, 51, 52, 53, 54},
    {FrontFace}, 1],
   M[{3, 6, 9, 2, 5, 8, 1, 4, 7, 54, 53, 52, 10, 11, 12, 13, 14,
     15, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36,
     37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 18, 17, 16},
    {BackFace}, 1],
   M[{13, 2, 3, 22, 5, 6, 31, 8, 9, 12, 21, 30, 37, 14, 15, 16,
     17, 18, 11, 20, 29, 40, 23, 24, 25, 26, 27, 10, 19, 28, 43, 32, 33, 34, 35,
     36, 46, 38, 39, 49, 41, 42, 52, 44, 45, 1, 47, 48, 4, 50, 51, 7, 53, 54},
    {LeftFace}, 1],
   M[{1, 2, 48, 4, 5, 51, 7, 8, 54, 10, 11, 12, 13, 14, 3, 18, 27,
     36, 19, 20, 21, 22, 23, 6, 17, 26, 35, 28, 29, 30, 31, 32, 9, 16, 25, 34,
     37, 38, 15, 40, 41, 24, 43, 44, 33, 46, 47, 39, 49, 50, 42, 52, 53, 45},
    {RightFace}, 1]
  };
```

Program 3

```
Clear[s, M, T]; TC = 0;
M /: M[a1_, {w1___}, m1_] ** M[a2_, {w2___}, m2_] := M[a1[[a2]], {w1, w2}, m1 + m2];
M /: Inverse[M[a_, w_, m_]] := M[Ordering[a], -Reverse[w], m];
Feed[M[Range[n], __]] := Null;
Feed[M[a_, {w___}, m_]] := Module[
   {modified = False, i, j, sij, k, l, skl},
   For[i = 1, a[[i]] == i, ++i]; j = a[[i]];
   If[Head[sij = s[i, j]] === Integer,
    (* then *) If[m ≥ T[sij][[3]] || False,
     Feed[ReplacePart[Inverse[T[sij]] ** M[a, {w}, m], {-sij, w}, 2]],
     modified = True; T[s[i, j] = ++TC] = M[a, {w}, m];
     Feed[ReplacePart[Inverse[M[a, {w}, m]] ** T[sij], -{w, -sij}, 2]]
    ],
    (* else *) modified = True; T[s[i, j] = ++TC] = M[a, {w}, m]
   ];
   If[modified,
    sij = s[i, j]; Do[
     If[Head[skl = s[k, l]] == Integer,
      Feed[ReplacePart[T[sij] ** T[skl], {sij, skl}, 2]];
      Feed[ReplacePart[T[skl] ** T[sij], {skl, sij}, 2]]
     ],
     {k, n}, {l, n}
    ]
   ]
  ];
Images[i_] := Prepend[Select[Range[n], Head[s[i, #]] === Integer &], i];
MoveCount[i_, i_] := 0;
MoveCount[i_, j_] := T[s[i, j]][[3]];

Dynamic[{TC, Images /@ Range[n], Sum[Total[MoveCount[i, #] & /@ Images[i]], {i, n}]},
 UpdateInterval → 1]
```

```
{126 328,
 {{1, 3, 7, 9, 10, 12, 13, 15, 16, 18, 28, 30, 31, 33, 34, 36, 37, 39, 43, 45, 46, 48, 52, 54},
  {2, 8, 19, 21, 25, 27, 38, 40, 42, 44},
  {3, 7, 9, 12, 13, 15, 16, 18, 28, 30, 31, 33, 34, 36, 37, 39, 43, 45, 46, 48, 54},
  {4}, {5}, {6}, {7, 9, 12, 13, 15, 16, 28, 30, 31, 33, 34, 36, 37, 39, 43, 45, 46, 48},
  {8, 19, 21, 25, 27, 38, 40, 42, 44},
  {9, 15, 16, 28, 30, 31, 33, 34, 36, 37, 39, 43, 45, 46, 48}, {10}, {11}, {12},
  {13}, {14}, {15}, {16}, {17}, {18}, {19, 21, 25, 27, 38, 40, 42, 44}, {20},
  {21, 25, 27, 38, 40, 42, 44}, {22}, {23}, {24, 29, 32, 35, 47, 51}, {25}, {26},
  {27, 38, 40, 42, 44}, {28, 30, 31, 33, 34, 36, 37, 39, 43, 45, 46, 48}, {29, 32, 35, 47},
  {30, 31, 33, 34, 36, 37, 39, 45, 48}, {31}, {32, 35, 47}, {33, 34, 36, 39, 45, 48},
  {34}, {35}, {36}, {37}, {38}, {39}, {40}, {41}, {42}, {43}, {44}, {45},
  {46}, {47}, {48}, {49}, {50}, {51}, {52}, {53}, {54}}, 156 105 796 449}
```

The Order of the Group

```
Timing[
  (Feed[#]; Product[Length[Images[i]], {i, n}]) & /@ Generators
]
```

(Aborted after about 18 hours).