Kohno: Space of conformal blocks and local systems on configuration spaces

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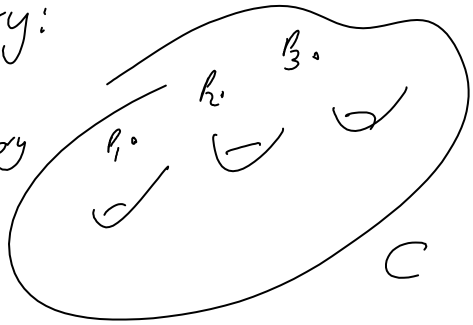
Conformal field theory:

A conformal field theory associates to C a finite dim complex vector space, "the space of conformal blocks".



$\mathfrak{g}$: a semi-simple Lie algebra $/\mathbb{C}$ $\left[\begin{array}{l}\text{for simplicity,}\\ G = sl(2,\mathbb{C})\end{array}\right]$

$$\hat{\mathfrak{g}} := \mathfrak{g} \otimes \mathbb{C}((\xi)) \oplus \mathbb{C}c$$

K a positive integer "the level"

$0 \leq \lambda \leq K$, $\lambda \in \mathbb{Z}$, V_λ: irrep of $\mathfrak{g}$ of height wt λ

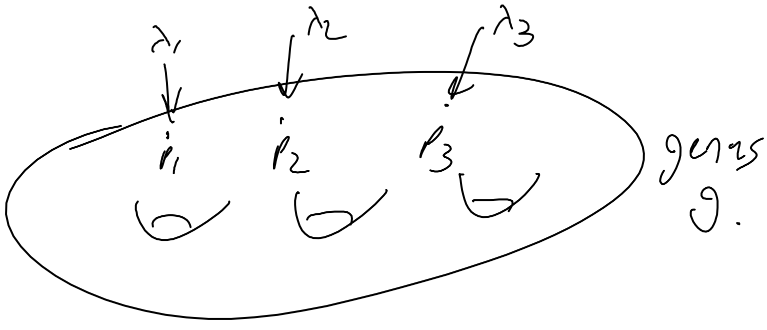
$Hv = \lambda v$, $Ev = 0$

$$\hat{\mathfrak{g}} = \mathfrak{n}_+ \oplus \mathfrak{n}_0 \oplus \mathfrak{n}_-$$

verma module $M_\lambda = U(\mathfrak{n}_-)(V_\lambda)$, $\mathfrak{n}_+ V_\lambda = 0$

$\mathcal{H}_\lambda$: The irreducible $\hat{\mathfrak{g}}$-module obtained as a quotient of M_λ

$0 \leq \lambda_i \leq K$



$\mathcal{H}_{p,\lambda}$

$\quad\shortparallel$

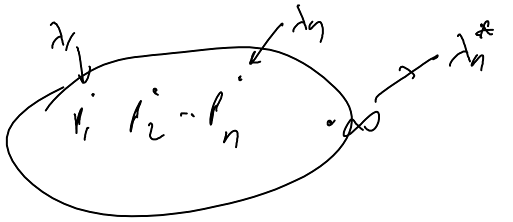
$\mathcal{H}_{\lambda_1} \otimes \mathcal{H}_{\lambda_2} \otimes \ldots \otimes \mathcal{H}_{\lambda_n} \Big/ \mathfrak{g} \otimes M_{p_1 \ldots p_n}$

[temporarily need to fix local coords near $p_1 \ldots p_n$]

meromorphic functions on C with poles at worst at $p_1 \ldots p_n$

$\mathcal{H}_{p,\lambda}$ depends on the complex structure and on the choice of local coords. So it defines a vector bundle on $\mathcal{M}_{g,n}$. It admits a natural projectively flat connection.

$X_n = \{(z_1 \cdots z_n) \in \mathbb{C}^n : z_i \neq z_j\}$



In this case $\mathcal{H}_{p,\lambda}$ is a quotient of

$$V_{\lambda_1} \otimes \ldots \otimes V_{\lambda_n} \otimes V^*_{\lambda_{n+1}} / g$$

The connection is the KZ connection:

$$w = \sum_{i<j} \frac{1}{k+2} \Omega_{ij} \, d \log(z_i - z_j)$$

$dw = 0$

$w \wedge w = 0$

casimirs: $\sum I_\mu \otimes I_\mu$ on i & j components.

Let $\lambda = \lambda_1 + \ldots + \lambda_n \ldots$

Varchenko - Schechtman and others construct horizontal sections of the KZ connection using configuration spaces:

$$X_{n,m} \longrightarrow X_{n+m}$$

$\check{x}_n$