Fock: Quantification des varietes amassees

June-19-09 4:05 AM

Cluster variety:

glued of elementary blocks:

$$((\mathbb{C}^*)^N, \epsilon) \quad \text{where } \epsilon \in M_{N\times N}(\mathbb{Z})$$

glued by elementary transformations: "mutations"

X-version

$$\varphi_k^x (x_1 \ldots x_N) \to (x_1' \ldots x_N')$$

$$x_i' = \begin{cases} x_i^{-1} & \text{if } i=k \\ x_i(1+x_k)^{\epsilon_{ik}} & \text{if } \epsilon_{ik} \geq 0 \end{cases}$$

A - version

$$\varphi_k^a (a_1 \ldots a_n) \to (a_1' \ldots a_n')$$

similar but different.

$$\text{with} \quad \epsilon_{ij}' = \begin{cases} -\epsilon_{ij} & \text{if } i=k \text{ or } j=k \\ \vdots \end{cases}$$

get

$$X(\epsilon) \xleftarrow[x_i = \prod_j a_j^{\epsilon_{ij}}]{} A(\epsilon)$$

This is an isomorphism if $\det \epsilon \neq 0$.

Running Example $\quad \epsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad \mathbb{C}^* = \left\{ \begin{pmatrix} a \\ b \end{pmatrix} \right\}$

Type A

$$\begin{pmatrix} a \\ b \end{pmatrix}_{\epsilon} \longrightarrow \begin{pmatrix} \frac{1+b}{a} \\ b \end{pmatrix}_{-\epsilon} \longrightarrow \begin{pmatrix} \frac{1+b}{a} \\ b^{-1} + a^{-1} + b^{-1}a^{-1} \end{pmatrix}_{\epsilon}$$

$$\left(\frac{1+a}{\ldots} \right) \quad \left(1+a \right) \quad //$$

A type

$$\Bigg\{ \longrightarrow \underset{-\epsilon}{\begin{pmatrix} \frac{1+a}{b} \\ b^{-1}+a^{-1}+b^{-1}a^{-1} \end{pmatrix}} \longrightarrow \underset{\epsilon}{\begin{pmatrix} \frac{1+a}{b} \\ a \end{pmatrix}} \longrightarrow \underset{-\epsilon}{\begin{pmatrix} b \\ a \end{pmatrix}}$$

Type X

$$\Bigg\{ \begin{pmatrix} x \\ y \end{pmatrix} \longrightarrow \begin{pmatrix} x^{-1} \\ y(1+x) \end{pmatrix} \quad \cdots \quad \longrightarrow \begin{pmatrix} y \\ x \end{pmatrix}$$

Poisson structure: $\{x_i, x_j\} = \epsilon_{ij} x_i x_j$

pre-Symplectic structure: $\omega = \sum_{ij} \epsilon_{ij} \frac{da_i \wedge da_j}{a_i a_j}$

So X type cluster varieties have a poisson structure,
A —||— pre-symplectic —||—.