A new w-formula for Alexander

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Given a directed band graph G in $\mathbb{R}^3$, let K be its thickening; assume it is a knot.
[So G is the core of a Seifert surface of K]. Assume G is already projected to the plane.

Let i,j enumerate the edges of G. Let L be the "linking matrix" of G:

$$L_{ij} := \begin{pmatrix} \text{the number of times edge } i \\ \text{goes over edge } j\text{, signs counted} \end{pmatrix}$$

Let $S^{\pm}$ be the "structural matrices" of G:

$$(S=S^+)_{ij} = \begin{pmatrix} \text{In the path in } \partial G \text{ from the} \\ \text{left head of } i \text{ to the right} \\ \text{head of } i\text{, the signed incidence} \\ \text{of } j. \quad [S_{ii} \text{ is included}] \end{pmatrix}$$

S^-_{ij} is the same, with heads replaced by tails.

Comment: It should be easy to compute L & S for the Seifert surfaces associated with braid presentations.

There ought to be a formula for $A(K)$ in terms of L and S. It should involve $\mathrm{tr}(I-LS)^{-1}$.