Why not Duflo?

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10:51 AM

Group-Ring statement. There exists $\omega^2 \in \mathrm{Fun}(\mathfrak{g})^G$ so that for every $\phi, \psi \in \mathrm{Fun}(\mathfrak{g})^G$ (with small support), the following holds in $\hat{\mathcal{U}}(\mathfrak{g})$: (shhh, $\omega^2 = j^{1/2}$)

$$\iint_{\mathfrak{g}\times\mathfrak{g}} \phi(x)\psi(y)\omega^2_{x+y}e^{x+y} = \iint_{\mathfrak{g}\times\mathfrak{g}} \phi(x)\psi(y)\omega^2_x\omega^2_y e^x e^y.$$

Convolutions and Group Rings (ignoring all Jacobians). If G is finite, $(\mathrm{Fun}(G), \star) \cong (\mathbb{R}G, \cdot)$ via $T : f \mapsto \sum f(a)\tau(a)$. For Lie $\mathfrak{g}$ and G,

$$\begin{tikzcd} (\mathfrak{g},+) \ni x \arrow[r, "\tau"] \arrow[d, "\exp"] & e^x \in \hat{\mathcal{S}}(\mathfrak{g}) \arrow[d, "\chi"] \\ (G,\cdot) \ni e^x \arrow[r, "\tau"] & e^x \in \hat{\mathcal{U}}(\mathfrak{g}) \end{tikzcd} \quad \text{so} \quad \begin{tikzcd} \psi \in \mathrm{Fun}(\mathfrak{g}) \arrow[r, "T"] \arrow[d, "\Phi^{-1}"] & \hat{\mathcal{S}}(\mathfrak{g}) \arrow[d, "\chi"] \\ \mathrm{Fun}(G) \arrow[r, "T"] & \hat{\mathcal{U}}(\mathfrak{g}) \end{tikzcd}$$

with $T\psi = \int \psi(x)e^x dx \in \hat{\mathcal{S}}(\mathfrak{g})$ and $T\Phi^{-1}\psi = \int \psi(x)e^x \in \hat{\mathcal{U}}(\mathfrak{g})$. Given $\psi_i \in \mathrm{Fun}(\mathfrak{g})$ compare $\Phi^{-1}(\psi_1)\star\Phi^{-1}(\psi_2)$ and $\Phi^{-1}(\psi_1\star\psi_2)$ in $\hat{\mathcal{U}}(\mathfrak{g})$:

$$\star \text{ in } G: \iint \psi_1(x)\psi_2(y)e^x e^y \qquad \star \text{ in } \mathfrak{g}: \iint \psi_1(x)\psi_2(y)e^{x+y}$$

$$\int f(x)e^x dx = \sum_{n=0}^{\infty} \frac{1}{n!} \sum f(x)\cdot x^n$$

$$\int f(t)t^n\alpha^n = \alpha^n \int f(t)t^n.$$

<u>claim</u> If $\int f(t)p(t) = 0$ for all polynomials p, then $f \equiv 0$.