Trieste Day 4 Handout

May-20-09
10:48 AM

u: here and now.
w: On Monday.
u⟷w Some is done,
Some not yet; no talk plans

Knotted Trivalent Graphs, Tetrahedra and Associators

Trieste Day 4 handout, May 2009 (based on HUJI–001116)

Dror Bar–Natan

I hope to work
on the v case
after completing
the w-warmup.

$(A\otimes B)\otimes C \xrightarrow[\text{in } U(\mathfrak{g})^{\otimes 3}]{\Phi} A\otimes(B\otimes C)$

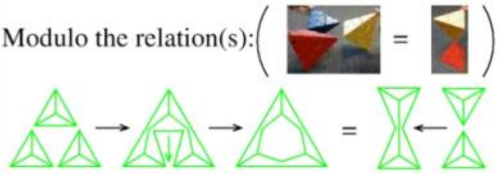
$((A\otimes B)\otimes C)\otimes D \xrightarrow{(\Delta 11)\Phi} (A\otimes B)\otimes(C\otimes D) \xrightarrow{(11\Delta)\Phi} A\otimes(B\otimes(C\otimes D))$

$((A\otimes B)\otimes C)\otimes D \xrightarrow{\Phi\otimes 1} (A\otimes(B\otimes C))\otimes D \xrightarrow{(1\Delta 1)\Phi} A\otimes((B\otimes C)\otimes D) \xrightarrow{1\otimes\Phi} A\otimes(B\otimes(C\otimes D))$

Φ is an "associator"

$(\Phi\otimes 1)\cdot(1\Delta 1)\Phi\cdot(1\otimes\Phi) = (\Delta 11)\Phi\cdot(11\Delta)\Phi$

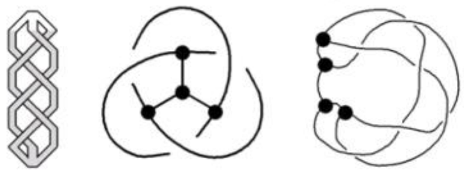
"The Drinfel'd Pentagon"

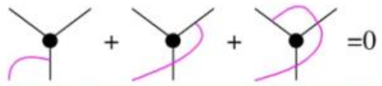
Modulo the relation(s): 

(+more)

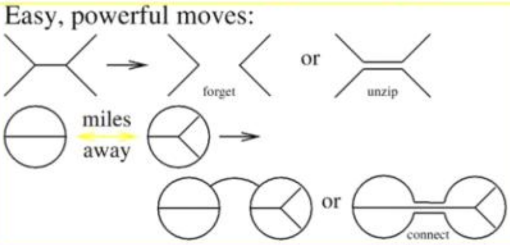
Claim. With $\Phi := Z(\triangle)$, the above relation becomes equivalent to the Drinfel'd's pentagon of the theory of quasi Hopf algebras.

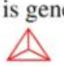
Extend to Knotted Trivalent Graphs (KTG's):

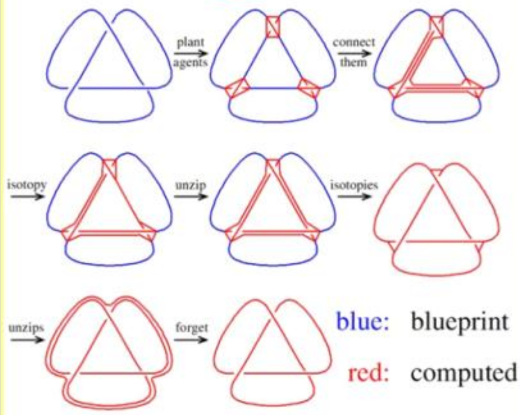


Need a new relation: 

Easy, powerful moves:



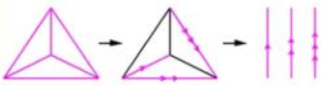
Using moves, KTG is generated by ribbon twists and the tetrahedron  :



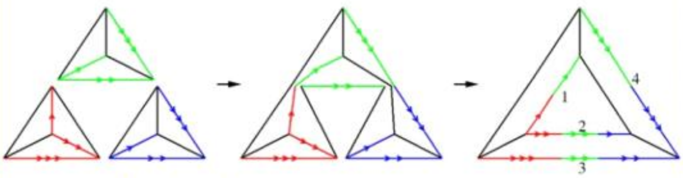
blue: blueprint

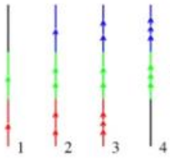
red: computed

Proof.

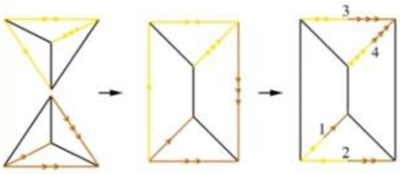


$:= \Phi \in \mathcal{A}(\uparrow_3)$





$= (\Phi\otimes 1)\cdot(1\otimes\Delta\otimes 1)(\Phi)\cdot(1\otimes\Phi) \in \mathcal{A}(\uparrow_4)$



$= (\Delta\otimes 1\otimes 1)(\Phi)\cdot(1\otimes 1\otimes\Delta)(\Phi) \in \mathcal{A}(\uparrow_4)$

What are associators good for? 0. Construct expansions.
1. Quantum group and the Kohno-Drinfel'd theorem, also heavily used in knot theory.
2. Etingof-Kazhdan quantization of Lie-bialgebras.
3. Tamarkin's proof of Kontsevich's deformation quantization theorem.
4. Alekseev-Torossian's study of Kashiwara-Vergne.
5. Maybe more?

See more at http://www.math.toronto.edu/~drorbn/Talks/Trieste-0905

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