

Cameron Gordon Day 1

Thm (Alexander)  $S^3$  is irreducible; that is, every smooth  $S^2$  bounds a ball.

Def  $M$  is prime if  $M = M_1 \# M_2 \Rightarrow M_1 = S^3$  or  $M_2 = S^3$

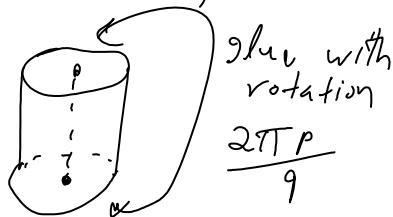
Thm A prime  $M^3$  is irreducible or  $M \cong S^1 \times S^2$

Thm (Kneser, Milnor) Unique factorization into primes for arbitrary  $M^3$ 's.

Def A 2-disk  $(D, \partial D) \subset (M, \partial M)$  is essential if  $\partial D$  does not bound a disk in  $\partial M$

Thm (Bonahon) If  $M$  is irreducible, then there is a submanifold that contains  $\partial M$  and (up to isotopy) all essential boundary disks in  $M$ .

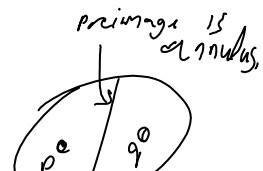
A Seifert Fiber Space (SFS):  $M = \cup \text{circles}$ , locally like



The space of fibers of an SFS is a surface  $F$ .

JST for knot complements. Let  $M_K$  be the knot complement of  $K \subset S^3$ . There are four possibilities:

1.  $K = V$ ,  $M_K = S^1 \times D^2 \supset \text{ess. } D^2$
2.  $K = T_{p/q}$   $M_K$  is an SFS w/



(p / i)

3. Satellite knots have an essential  $T^2$  in their complements.

This is iff  $\nabla$ . If  $M_K$  has an essential  $T^2$ ,  $K$  is a satellite.

4. otherwise  $M_K$  is hyperbolic.