Fenn in Trieste

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A funny quandle: starting from a group, set $\boxed{g \wedge h := hg^{-1}h.}$ satisfies:

Aside: Racks only need to satisfy 2 & 3

1. $a \wedge a = a$
2. Given a, b $\exists$ a unique c s.t. $c \wedge b = a$
3. $(a \wedge b) \wedge c = (a \wedge c) \wedge (b \wedge c)$

Question Can one classify all quandles that arise from groups?

Added July 19, 2015: Possibly this is answered in Wada's "group invariants for links".

The Alexander quandle. $\Lambda = \mathbb{Z}[t, t^{-1}]$,

$$a \wedge b = ta + (1-t)b$$
$$a \wedge \bar{b} := t^{-1}a + (1-t^{-1})b$$

Verifying Axiom 3 for $a \wedge b := ba^{-1}b$:

$$(a \wedge b) \wedge c = c\, b^{-1} a b^{-1} c \quad ✓$$
$$(a \wedge c) \wedge (b \wedge c) = (ca^{-1}c) \wedge (cb^{-1}c) = cb^{-1}\cancel{c c^{-1}} a \cancel{c^{-1} c} b^{-1} c$$

Interesting comment - in a general quandle we may have a^b=a yet not b^a=b. This will never happen in a quandle coming from a group as in a group the "a commutes with b" relation is symmetric.